Power Sharing Correction in Angle Droop Controlled Inverter Interfaced Microgrids

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***Abstract*—Power sharing between angle-droop controlled inverter-based sources largely depends on the line impedances and choice of droop coefficients. Simple power correction methods such as set-point correction and droop coefficient modification work satisfactorily for specific network topologies and require only limited amount of communication. However, their performance can be inadequate for microgrids with different topologies. In this work, we propose a topology-independent power sharing correction technique based on inter-node communications in order to eliminate the power sharing errors between inverters. We analyse stability and convergence of the proposed solution and present simulation results. Finally, we study the robustness of frequency-droop and angle-droop controlled systems with respect to unknown impedances and parameter uncertainties.**

***Index*—Angle droop control, frequency droop control, power sharing error, consensus protocol, parameter mismatches, power converters, microgrids.**

I. INTRODUCTION

As distributed and renewable power generation become an increasingly common phenomenon, parts of the electricity network with high supply to demand ratio have the opportunity to operate in isolation from the main grid. Such isolated parts of the electricity networks are generally known as microgrids [1]. Since most of the renewable energy sources require an inverter interface to connect to the AC grid, microgrids based on renewable energy can be considered as low inertia networks. Achieving synchronization, voltage and frequency stability in such low-inertia systems is a challenging task.

Inverter-interfaced microgrid design should be able to compensate for the loss of grid (scarcity in inertia) and also provide reliable power quality. Ideally, all the sources in the microgrid should be able to compensate for the loss of grid, i.e. regulate the voltage (V) and frequency (f) within permissible levels and avoid a single point-of-failure. Additionally, due to intermittent generation profiles of renewable energy sources, limited energy storage, and constrained inverter capabilities, power sharing between inverters is a desirable property.

Motivated by synchronous generators, power sharing between inverter based sources through *droop control* was initially proposed in [2]. Although droop control is usually implemented on frequency and real power ($P - f$ droop), various modifications of droop control have been studied. One well known variant of droop control is the *angle-droop* first proposed in [3]. Implemented only on real power (P - for highly inductive networks), this scheme is motivated by the fact that small angle differences will cause a change in the power sharing between the sources. Therefore, each inverter is controlled to change its phase angle (δ) according to its real power (P) output. Depending on the network characteristics the angle-droop is also modified to control the real or reactive power flow in networks [4]. Although the implementation is challenging, angle-droop controlled inverter systems provide better stability margin [5], [6]. Unlike frequency-droop, angle-droop causes zero frequency deviation. However, power sharing between angle-droop controlled inverters is affected by network impedance distribution [4], [5].

Contributions: In this work, we analyse the power sharing properties of angle-droop controlled systems under different network topologies. We identify the limitations of implementation and power sharing correction techniques discussed in the literature [4], [5]. Inspired by consensus-based frequency restoration [7] and consensus-based droop control techniques [8], we propose a topology-independent inter-node communications based power sharing correction technique to eliminate the power sharing errors. We also perform a convergence analysis to demonstrate the stability and performance of the proposed technique. Finally, we analytically compare the performance of frequency-droop and angle-droop controlled systems, particularly with respect to parameter uncertainties.

Paper Organization: The fundamentals of power sharing and angle-droop control are discussed in Section II. A consensus based power sharing technique is proposed and discussed in Section III. The frequency-droop and angle-droop controlled systems are compared in Section IV. Simulations results are presented in Section V followed by conclusion and discussions on future directions in Section VI.

II. ANGLE-DROOP CONTROL

Phasor power flow between the source $V_1\angle\delta_1$ and load $V_0\angle\delta_0$ shown in Figure 1 is given in (1).

$$\vec{S_{10}} = P_{10} + jQ_{10} = = \vec{V_1}\vec{I_{10}}^* = \vec{V_1}\left(\frac{\vec{V_1} - \vec{V_0}}{\vec{Z_{10}}}\right)^* \qquad (1)$$

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This work has been partly funded by a Linkage Grant supported by the Australian Research Council and Senergy Australia.

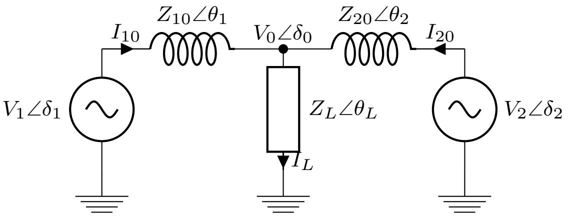

Fig. 1: Power flow within a two inverter system supplying a common load Z_L.

For inductive lines the X/R ratio is very large, based on which (1) can be approximated by [2]

$$P_{10} \approx \frac{V_1 V_0 \sin(\delta_1 - \delta_0)}{X_{10}} \tag{2}$$

$$Q_{10} \approx \frac{V_1 (V_1 - V_0 \cos(\delta_1 - \delta_0))}{X_{10}} \tag{3}$$

For small angle differences $(\delta_1 - \delta_0)$ the trigonometric functions in (2) and (3) can be approximated as $\cos(\delta_1 - \delta_0) \approx 1$ and $\sin(\delta_1 - \delta_0) \approx (\delta_1 - \delta_0)$, yielding

$$(\delta_1 - \delta_0) \cong \chi_{10} P_{10} \tag{4}$$

$$(V_1 - V_0) \cong \chi_{10} Q_{10} V_0 \tag{5}$$

with $\chi_{ij} \triangleq \dfrac{X_{ij}}{V_i V_j}$. As seen in (4) and (5), the amount of real power flowing between any two nodes can be controlled by changing the angle δ between them and the amount of reactive power flowing can be controlled through changes in voltage V. This forms the basis of angle and voltage-droop controllers:

$$\delta_i = \delta_{i,\text{rated}} - m_i \times (P_i - P_{i,\text{rated}}) \tag{6}$$

$$V_i = V_{i,\text{rated}} - b_i \times (Q_i - Q_{i,\text{rated}}) \tag{7}$$

We refer to [9] for inverter modelling and Figure 2 of [6] for angle-droop implementation. Note that, the implementation of angle-droop control requires a common clock signal for time synchronization at all inverters and the angle measurements are with respect to a common angular frequency, typically ω_{rated} [4]. Here, the values P_i and Q_i are obtained from their instantaneous values using a low pass filter with cut-off f_c [5].

A. Power sharing in angle-droop controlled systems

In the previous section we have established the power flow relationship between a source and a load. As mentioned in Section I, the main aim of the droop controller is to provide robust power sharing capabilities. Therefore, it is important to see how the power sharing between the sources is modified using angle-droop control. From Figure 1 and calculations made earlier, the angle differences between the sources and the load are given by

$$(\delta_1 - \delta_0) \cong \chi_{10} P_{10}, \quad (\delta_2 - \delta_0) \cong \chi_{20} P_{20} \tag{8}$$

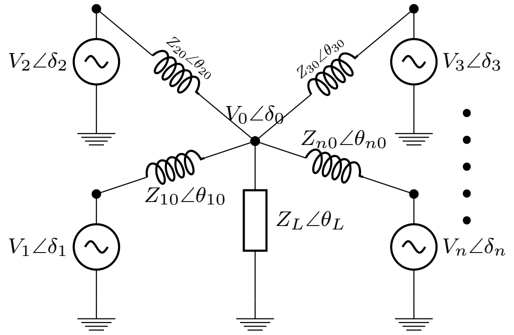

Fig. 2: Star topology - n droop controlled inverter systems supplying a common load Z_L. Each inverter is represented by a phasor $V_i\angle\delta_i$ and is connected to the load ($V_0\angle\delta_0$) through an output impedance $Z_{i0}\angle\theta_{i0}$.

Assuming that Sources 1 and 2 are operated on angle-droop control and also assuming $\delta_{i,\text{rated}} = m_i P_{i,\text{rated}}$, the angle between the two sources can be represented by (9). Solving equations (8) and (9) together yields (10).

$$\delta_1 - \delta_2 = m_2 P_{20} - m_1 P_{10} \tag{9}$$

$$P_{10} \left(\chi_{10} + m_1\right) = P_{20} \left(m_2 + \chi_{20}\right) \tag{10}$$

It can be seen from (10) that only a choice of $m_i >> \chi_{i0}$ will result in the desired power sharing ratio $m_1 P_{10} \approx m_2 P_{20}$.

B. Load voltage communication

In situations where the droop coefficients are small, there can be an undesirable power sharing error. Then, a coordination control is necessary to reduce the power sharing error between sources. With periodic communication of load voltage V_0 and assuming we have a good estimate of X_{i0}, we can change the value of $\delta_{i,\text{rated}}$ of each inverter to be equal to the associated deviation $\chi_{i0} P_{i0}$. This choice of $\delta_{i,\text{rated}}$ will counteract the power sharing error and ensure proper power sharing [4]. Alternatively, if we have the flexibility to choose $\delta_{i,\text{rated}} = P_{i,\text{rated}} = 0$, power sharing of the system shown in Figure 1 can be restored by using modified droop coefficients (11). The droop equations implemented at each inverter, in this case, are given by (12).

$$m_{1,\text{new}} = m_{1,\text{desired}} - \chi_{10}, \quad m_{2,\text{new}} = m_{2,\text{desired}} - \chi_{20} \tag{11}$$

$$\begin{aligned} \delta_1 &= -\left(m_{1,\text{desired}} - \chi_{10}\right) P_{10}, \\ \delta_2 &= -\left(m_{2,\text{desired}} - \chi_{20}\right) P_{20} \end{aligned} \tag{12}$$

Performing calculations similar to those made earlier will yield

$$\begin{aligned} \left(m_{2,\text{desired}} - \chi_{20}\right) P_{20} - \left(m_{1,\text{desired}} - \chi_{10}\right) P_{10} \\ = \chi_{10} P_{10} - \chi_{20} P_{20} \end{aligned} \tag{13}$$

The error terms in (13) cancel out and the desired power sharing is guaranteed as follows:

$$m_{1,\text{desired}} P_{10} = m_{2,\text{desired}} P_{20} \tag{14}$$

1) Other network configurations: As we have seen earlier, the modified droop coefficients will ensure desired power sharing in a network with star topology (as shown in Figure 2). We next investigate how the correction technique performs

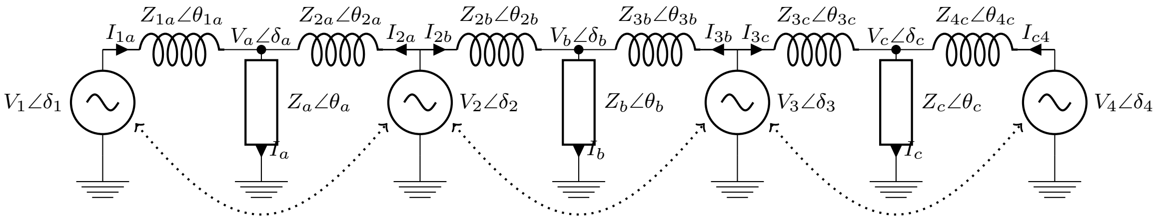

Fig. 3: Four inverter three load microgrid. Dotted lines represent communication links between inverters using which each inverter i communicates its output power P_i to its neighboring inverters $\mathcal{N}_i$, see section III.

under different network configurations. As an example, the power sharing between inverters in a four inverter system shown in Figure 3 is considered where the difference between the voltage angles is

$$\delta_1 - \delta_2 = \chi_{1a} - \chi_{2a} \tag{15}$$

$$\delta_2 - \delta_3 = \chi_{2b} - \chi_{3b} \tag{16}$$

$$\delta_3 - \delta_4 = \chi_{3c} - \chi_{4c} \tag{17}$$

Given that each source implements angle-droop, we also have

$$\delta_1 - \delta_2 = m_2(P_{2a} + P_{2b}) - m_1 P_{1a} \tag{18}$$

$$\delta_2 - \delta_3 = m_3 P_{3b} - m_2(P_{2a} + P_{2b}) \tag{19}$$

$$\delta_3 - \delta_4 = m_4 P_{4c} - m_3(P_{3b} + P_{3c}) \tag{20}$$

In this case, choosing $\delta_{i,\text{rated}}$ or $m_{i,\text{new}}$ to counteract the power sharing error is prohibitively difficult due to the unavailability partial power terms. Measuring the partial power terms for $\delta_{i,\text{rated}}$ correction becomes impractical and the also load voltage feedback method (modifying m_i to $m_{i,\text{new}}$) will not ensure accurate power sharing. A communication based central controller can be used to implement a power sharing correction technique as shown in [4], but such a system is susceptible to a single point-of-failure. In summary, choosing a really large m_i or modifying m_i based on load voltage communication is not suitable when $X_{ij}/(V_i V_j)$ is large as it can destabilize the system [4]. Also, obtaining the information on X_{ij} for all branches is very difficult, especially in case of large networks. Moreover, limited communication will not eliminate power sharing error for all configurations due to the unavailability of required information. Therefore, it is desirable to have a power sharing correction technique that requires sparse communication and ensure zero power sharing error.

III. POWER SHARING AS A CONSENSUS PROBLEM

In this section, we make use of inter-node communications to eliminate the power sharing error between inverters. Each inverter i communicates its output power P_i to the neighbouring inverters $\mathcal{N}_i$ as shown in Figure 3. Since, the angle-droop controller implementation requires P_i measurement, there is no need for any additional sensors in the network.

Preliminaries to graph theory: The (undirected) communications between inverters for the power sharing correction can be represented as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. $\mathcal{V}$ is the set of inverters and $\mathcal{E}$ is the set of edges which represent the communication links between inverters. We define the degree matrix $\mathcal{D} \triangleq \textbf{diag}\{\deg(1), \deg(2) \ldots \deg(\text{n})\}$, where n is the number of inverters in the system and deg(n) is the number of communication links connected to the n^{th} inverter. Adjacency matrix $\mathcal{A}$ represents the connection between inverters in the communication graph with $a_{ij} = a_{ji} = 1$ if the inverters i and j are connected, and $a_{ij} = a_{ji} = 0$ if they are not connected. Self loops are avoided, resulting in $a_{ii} = 0$ for any inverter i. We denote the communication graph Laplacian $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

$$\mathcal{A} = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \ldots & a_{nn} \end{bmatrix}, \mathcal{L} = \begin{bmatrix} L_{11} & \ldots & L_{1n} \\ \vdots & \ddots & \vdots \\ L_{n1} & \ldots & L_{nn} \end{bmatrix}$$

A. *Proposed power sharing correction technique*

Based on ideas from consensus theory [10], we modify the droop equations adding an integral term p_i as shown in (21) and (22) to facilitate power sharing error correction.

$$\delta_i = \delta_{i,\text{rated}} - m_i \times (P_i - P_{i,\text{rated}}) - p_i \tag{21}$$

$$\dot{p_i} = k_i \sum_{j \in \mathcal{N}_i} (m_i P_i - m_j P_j) = k_i \sum_{j=1}^{n} L_{ij} (m_i P_i - m_j P_j) \tag{22}$$

Convergence Analysis: Modifying (21) and (22) and representing in vector notation will result in the closed loop system with a communication overlay:

$$\text{Closed loop system, } \dot{\mathbf{p}} = -\mathcal{K}\mathcal{L}\left(\mathbf{p} + \boldsymbol{\delta} - \mathcal{M}\mathbf{P_r} - \boldsymbol{\delta_r}\right) \tag{23}$$

$$\text{where } \mathcal{K} = \textbf{diag}\{k_1, k_2 \ldots k_n\}, \mathcal{M} = \textbf{diag}\{m_1, m_2 \ldots m_n\}$$
$$\text{are matrices and } \boldsymbol{p} = (p_1, \ldots, p_n)^T, \boldsymbol{\delta} = (\delta_1, \ldots, \delta_n)^T$$
$$\boldsymbol{\delta_r} = (\delta_{1,\text{rated}}, \ldots, \delta_{n,\text{rated}})^T, \mathbf{P_r} = (P_{1,\text{rated}}, \ldots, P_{n,\text{rated}})^T$$

are vectors. All (except first) eigenvalues of the Laplacian $\mathcal{L}$ in (23) are increasing, therefore, all (except first) eigenvalues of $-\mathcal{L}$ are decreasing [10]. The eigenvalue $\lambda_1 = 0$ of $\mathcal{L}$ corresponds to the eigenvector $\mathbf{1} = (1, \ldots, 1)^T$ since $\mathbf{1}$ belongs to the nullspace of $\mathcal{L}$. This means that the equilibrium of the system (23) is of the form $\mathbf{p}^* = \alpha\mathbf{1} + \boldsymbol{\beta}$ with $\alpha = \frac{1}{n}\sum_i^n m_i P_i$ and bias $\boldsymbol{\beta} = \mathcal{M}\mathbf{P_r} + \boldsymbol{\delta_r} - \boldsymbol{\delta}$, according to the *average consensus theorem* [10]. This ensures that the new augmented term p_i will restore desired power sharing and does not destabilize the droop controlled system, given the angle-droop system is stable in itself. As we can see from the analysis, the network configuration does not affect the restoring capability of the proposed algorithm. The speed of convergence is determined by the *algebraic connectivity* of the communication graph (i.e., second smallest eigenvalue λ_2 of the Laplacian matrix $\mathcal{L}$), see [11] for more details. Therefore, matrix $\mathcal{K}$ provides us with the degree of freedom to achieve the desired speed of convergence. It should be noted that this method will ensure proper power sharing only if the communication path traversed from node 1 to node n contains all the nodes in-between, i.e., if the graph is well-connected.

IV. ANGLE-DROOP VS FREQUENCY-DROOP

As mentioned earlier, frequency-droop is another real power sharing control scheme. Based on similar principles as in angle-droop, this controller dynamically changes the angle by changing the frequency:

$$\omega_i = \omega_{i,\text{rated}} - m_{i,f} \times (P_i - P_{i,\text{rated}}) \tag{24}$$

$$m_{i,f} P_{i,\text{rated}} = m_{j,f} P_{j,\text{rated}} \tag{25}$$

It is known that irrespective of the line (inductive) impedance distribution, the frequency-droop controller (24) is capable of accurate power sharing given the droop coefficients are employed as per the feasibility criteria (25). However, a deviation in the overall system frequency can be stated as the major drawback of the frequency-droop controller. While angle-droop controller causes no frequency deviation, its complex implementation and lack of robustness to parameter uncertainties appear as limiting factors. Let us now analytically compare the performance of both the systems under frequency deviations. Since inverters lack inertia, frequency deviations can occur from various conditions such as bad inverter clocks, unsynchronized inverter interconnection, etc. As discussed earlier, the angle-droop system (6) is implemented in a rotating frame which is rotating at an angular speed of ω_{rated}. However, maintaining ω_{rated} at all inverters is a conservative assumption and very difficult to achieve at all times.

$$\delta_i = \int_0^t \omega_{i,\text{rated}} + \delta_{i,\text{rated}} - m_i \times (P_i - P_{i,\text{rated}}) \tag{26}$$

Any small perturbation that causes a difference between the rated frequencies $\omega_{i,\text{rated}}$ and $\omega_{j,\text{rated}}$ at the i^{th} and j^{th} inverters, respectively, will modify the angle between them as

$$\delta_i - \delta_j = \delta_{ij}(t) - m_i P_i + m_j P_j. \tag{27}$$

Here, $\delta_{ij}(t) = \int_0^t (\omega_{i,\text{rated}} - \omega_{j,\text{rated}})$ is an increasing ramp function that translates into a sawtooth function between 0 and 2π radians in the rotating frame, causing the power sharing between inverters i and j to vary proportional to $\sin \delta_{ij}(t)$. Therefore, we can conclude that the angle-droop controlled system is never stable no matter how small the value of $(\omega_{i,\text{rated}} - \omega_{j,\text{rated}})$ is. Measuring angles in such a scenario makes implementation prohibitively difficult. A supervisory control is therefore necessary for frequency correction in such situations. On the other hand, the frequency-droop controlled systems are robust to small parameter uncertainties. Real power sharing in a frequency-droop controlled system is independent of the distribution of line impedances (for inductive networks). We refer to [7] for stability and power sharing analysis of frequency-droop controlled systems and [12] for understanding their performance under parameter uncertainties.

V. SIMULATIONS AND DISCUSSION

To demonstrate the performance of our proposal, we simulated the four-inverter network shown in Figure 3 with parameters shown in Table I in MATLAB Simulink. These parameters are in line with relevant works [3] (with few modifications). Figure 4 (top) shows the performance of the simple angle-droop controlled system. It can be observed that there are large power sharing errors between sources although their droop coefficients are all equal. These deviations arise due to the disproportionate impedances between the sources and loads. The performance of the angle-droop controlled system combined with the proposed consensus based power correction algorithm is shown in Figure 4 (bottom). We have chosen large load changes to illustrate the ability of the controller to quickly regulate power. It can be clearly seen that the proposed power sharing correction technique ensures zero power sharing error irrespective of the line impedances and converges very quickly.

TABLE I: Simulation parameters - power sharing error

Parameter	**Value**
n, $V_{i,\text{rated}}$, $\omega_{i,\text{rated}}$	4, 415V, $2\pi 50$ rad/s
m_i, b_i, $\delta_{i,\text{rated}}$	$10^{-7} rad/W$, $10^{-4} V/Var$, 0 rad
$P_{i,\text{rated}}$, $Q_{i,\text{rated}}$	0 kW, 0 $kVar$
$[Z_{1a} \;\; Z_{2a} \;\; Z_{2b}]$	$j\omega_{i,\text{rated}}[1.8 \;\; 1.4 \;\; 1.6] \;\; 10^{-2}\Omega$
$[Z_{3b} \;\; Z_{3c} \;\; Z_{4c}]$	$j\omega_{i,\text{rated}}[1.3 \;\; 2 \;\; 1] \;\; 10^{-2}\Omega$
k_i, f_c	$10^4/s$, 10Hz

TABLE II: Simulation parameters - frequency mismatches

Parameter	**Value**
n, $V_{i,\text{rated}}$, $\delta_{i,\text{rated}}$	2, 415V, 0 rad
m_i, $m_{i,f}$, b_i	$10^{-7} rad/W$, $10^{-3} rad/Ws$, $10^{-4} V/Var$
$P_{i,\text{rated}}$, $Q_{i,\text{rated}}$	0 kW, 0 $kVar$
$[Z_{10} \;\; Z_{20}]$	$j\omega_{i,\text{rated}}[1.8 \;\; 1] \;\; 10^{-2}\Omega$
$\omega_{1,\text{rated}}$ $\omega_{2,\text{rated}}$, f_c	$2\pi(50.001)$ rad/s, $2\pi 50$ rad/s, 10Hz

A comparison between the performance of angle-droop and frequency-droop controller is presented in Figure 5. From Figure 5 (bottom) it can be seen that a small deviation in frequency on the first inverter $(\omega_{1,\text{rated}} - \omega_{2,\text{rated}}) = 2\pi(0.001)$ radians will cause a disturbance in the power sharing for angle-droop controlled systems and destabilize them. On the contrary, the frequency-droop controlled system is stable for small frequency differences as shown in Figure 5 (top). Also, the unequal line impedances do not show any effect on the power sharing between frequency-droop controlled systems.

VI. CONCLUSION AND FUTURE WORKS

In this work, we have shown the power sharing limitations of angle-droop controlled inverter based microgrids. Angle-droop controlled inverters will not be able to provide accurate

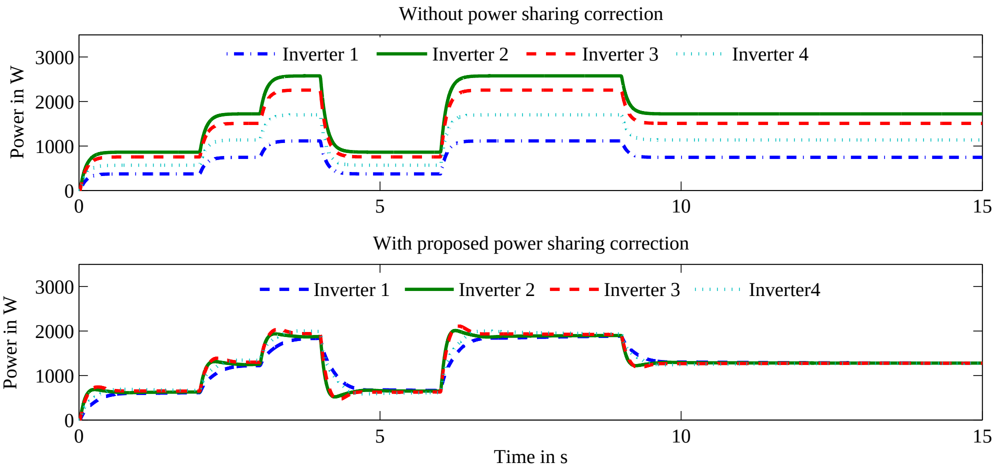

Fig. 4: Simulation results for power sharing between inverters showed in Figure 3 with parameters given in Table I (top) without any power sharing correction technique; (bottom) with the proposed consensus based power sharing correction technique. Load impedances $Z_a = Z_b = Z_c = 100\Omega_{\text{at time } t\,=\,0,\,4\text{s}}$; $50\Omega_{\text{at time } t\,=\,2,\,9\text{s}}$ and $33.3\Omega_{\text{at time } t\,=\,3,\,6\text{s}}$.

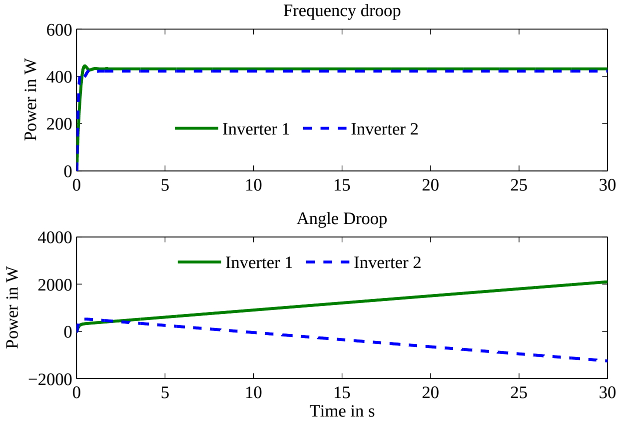

Fig. 5: Power sharing between inverters shown in Figure 1 with parameters given in Table II (top) frequency-droop controlled system; (bottom) angle-droop controlled system. $Z_L = 100\Omega$

proportional power sharing unless the line impedances are chosen according to their droop coefficients. Power correction techniques based on sparse communications, discussed in previous literature will not ensure proper power sharing and their implementation is difficult. To overcome these problems, we proposed a control technique where each inverter communicates with its neighbours and implement an integral control in addition to the angle-droop control. The implementation of this technique is fairly simple and relies only on information which is already available. It is shown, using simulations, that this integral controller will eliminate the power sharing errors between systems very efficiently without affecting the stability of the network. We have also analysed the behaviour of angle-droop controlled systems under parameter uncertainties. It is shown that the angle-droop controlled systems are not stable in the presence of frequency mismatches, unlike the frequency-droop controlled systems. Future research will consider the effect of communication latencies on the proposed technique. We also aim to extend the proposal to networks with higher R/X ratios and hybrid DC/DC, DC/AC microgrids.

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