A Market Mechanism for Electric Vehicle Charging Under Network Constraints

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***Abstract*—The increasing impact of electric vehicles on distribution networks can be alleviated by smart charging—the shifting of electric vehicle load to times when there is available capacity in the network. This work presents a market mechanism for smart charging that optimally allocates available charging capacity in a way that ensures network stability, while at the same time allowing vehicles to express individual preferences regarding their charging rates. Those who want higher rates can receive these, but must pay a higher price. The mechanism takes into account network-specific constraints such as total network load, voltage drop, and phase unbalance. However, since vehicles have differing impacts on these constraints, this leads to unequal access to the available resources (i.e., charging capacity), resulting in an unfair market. An additional constraint can be introduced to level the playing field for all users, but it leads to a reduction in aggregate performance. The mechanism is shown to be efficient and strategy-proof, so users cannot gain an unfair advantage by misrepresenting their preferences. A series of simulations demonstrate the mechanism's behavior and properties. The results open the door to multi-tiered user plans by demand response aggregators.**

***Index Terms*—Market mechanism, electric vehicles, demand response, network constraints, individual preferences.**

I. Introduction

ELECTRIFIED transport is being increasingly promoted around the world, and almost all major manufacturers have now released fully electric or plug-in hybrid electric vehicles for mainstream markets. The benefits of electric vehicles (EVs) are well understood (zero tailpipe emissions, suitability for use of renewable energy), but so too are the challenges of integrating them into electricity networks. An increasing body of research is now dedicated to examining the impacts of electric vehicles on distribution networks, and to finding ways to alleviate these impacts. This requires a strong understanding of the underlying network constraints, but it is important too to understand user behaviors and preferences. This paper aims to bridge the gap between these two domains by proposing a market mechanism for electric vehicle charging that not only respects the underlying network constraints, but also makes it possible for different user preferences to be incorporated in a way that is fair, efficient, and immune to cheating.

EVs have significant energy requirements and increasing uptake can lead to a number of undesirable consequences, such as thermal overload of network components, low voltages at sensitive locations of the network, and increased phase unbalance [1]–[4]. For public charging stations, careful siting and sizing of charge stations can help alleviate these impacts [5]. For home charging (in residential networks) early uptake is expected to be clustered in accordance with e.g., geography and demographics, with some neighborhoods experiencing much greater uptake than others [6]; nevertheless the location and amount of expected vehicle charging is difficult to predict and to plan for. A more feasible approach is to alleviate potential negative impacts of EV charging by scheduling charging at times when there is available capacity in the network, such as overnight. Methods achieving such load shifting are often classified as being either centralized or distributed.

So-called "centralized" methods aim to solve this problem by communicating relevant information to a central entity who then allocates available capacity as required. The optimization objective can vary. One way to achieve this is to minimize losses within the network [7], and the relationships between losses, load factor, and load variance are further explored using three different optimal charging schemes in [8]. Another way is to maximize the allocation of available charging capacity [2], [9]. A further advantage of centralized charge control is the possibility of using electric vehicle charging for reactive power compensation [10].

So-called "distributed" methods typically aim to solve this problem by allowing each vehicle (or charging unit) to make its own charging decisions. One increasingly common approach is to use local voltage measurements to estimate existing network loading levels [11], [12]. Other approaches include the use of game theory [13], sliding mode control [14], or via adaptive methods used also in communications [15].

However, a key factor in load shifting for EVs that is often overlooked – both in centralized and in distributed approaches – is the end user's preference. An owner may have changing requirements, and some owners will be willing to

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pay more for higher levels of service than others. Demand response aggregators may want to offer their customers multi-tiered plans (e.g., where high-paying customers have priority access to available charging capacity). Therefore an existing research gap is to address this issue: how can individual user preferences be incorporated in a way that accurately addresses individual user requirements, while still ensuring that general vehicle charging demands are met as well as possible for all users? How can this be done in a way that is fair, and does not allow a given user to cheat or otherwise manipulate the system?

Such questions apply to a variety of controllable loads, and the literature on demand response is extensive [16], [17]. When different user preferences are introduced, the problem typically becomes one of limited access to a shared resource by multiple users, and as a result market models are often applied (see [18] and [19]). The design of such models has led several authors to explore the application of mechanism design to demand response.

Samadi *et al.* propose a Vickrey-Clarke-Groves (VCG) based mechanism for demand side management: each user expresses a desire for scheduling of shiftable loads via a utility function, and the resulting mechanism maximizes the "social welfare", defined as the aggregate utility of all users minus the total energy cost [20]. In a related approach, Cao *et al.* use a back-and-forth approach between the utility and customers, where customers bid for their share of available power based on their needs and pay a price that reflects their share [21]. Gerding *et al.* and Robu *et al.* have applied mechanism design specifically to EVs: in their approach, users bid for power across time windows in which a vehicle is available for charging, and the mechanism allocates units of electricity according to communicated preferences in a manner that ensures truthfulness is preserved [22], [23].

However, most existing work does not address a key aspect of demand response: the constraints at the distribution level. Problems such as overload, under-voltage and phase unbalance impose limitations on what is possible in low voltage networks. As this paper shows, even the apparently simple task of ensuring a level playing field for all users turns out to be non-trivial.

In this work, we extend existing mechanism design based approaches to demand response by explicitly taking the underlying network constraints at the distribution level into account. While the mechanism we propose falls into the class of Groves mechanisms, which VCG also belongs to [20], one of the key differences to our approach is that the available resources (in our case charging capacity) are allocated in such a way that voltage and current levels are kept within desirable levels at all points in the distribution network. Instead of assuming that the "market operator" inherently knows what levels of charging are safe, we explicitly integrate the network constraints into the mechanism itself.

The mechanism proposed in this paper is further shown to be efficient and strategy-proof, and maximizes total welfare while ensuring that all bids are honest. Simulation results using a model of a real network show how this mechanism could be effectively implemented on a real system, which also opens the door to a multitude of indirect mechanism alternatives and multi-tiered user plans by demand response aggregators.

The rest of this paper is structured as follows: Section II summarizes an optimal charge allocation scheme that the rest of this paper builds on; Section III extends this scheme to additionally take individual user preferences into account via a market mechanism. Section IV describes why network constraints can lead to unfair market access. Section V presents simulation outcomes, while Section VI provides some discussion and Section VII concludes the paper.

II. Optimal Charging

The market mechanism for EV charging developed in this paper builds on a previously developed optimal charge allocation scheme [9] – due to space limitations, only a brief overview is presented here.

A. Problem Description and Notation

The goal is to charge all EVs in a radial distribution network served by one transformer. It is assumed that EVs' charging rates may be assigned within a continuous range by a central controller which knows the network topology and has access to key network parameters such as line and transformer limitations (as was recently demonstrated in a trial [24]). The rates are chosen not just for the present point in time, but for a series of discrete time intervals in a finite future window. However, the full set of charging rates for the shifting horizon is recalculated at discrete intervals, thus taking into account changes in underlying conditions (such as fluctuations in household demand or vehicle arrival/departure).

To simplify the problem and for scaleability, a DC-equivalent of the distribution network is assumed when determining key network constraints. This is a common assumption when the network is mainly resistive, which is true for most distribution networks. This linear approximation is further explored and justified in [9] and allows us to express charge allocation as a linear program.

Let $\mathcal{H}$ be the set of houses served by a single transformer in the network. Of these, a subset $\mathcal{K} \subset \mathcal{H}$ have EVs that are presently connected and need to be charged (K in total). Charging rates are chosen over the discretized charging horizon $\mathcal{T}$ having T intervals. The current at an individual household h at time t is denoted by $x_{h,t}$ (current due to household load) and $x_{k,t}$ (current due to vehicle load). Total current at household h at time t is:

$$x_{h,t}^{\text{tot}} = x_{h,t} + x_{k,t}$$

The network is modeled as a three-phase wye-connected system with individual households connecting single phase to neutral. The total current on a given phase is the sum of all currents at any points of connection on that phase:

$$x_{\phi,t} = \sum_{h \in \phi} x_{h,t}^{\text{tot}}, \quad \phi \in \{\phi_1, \phi_2, \phi_3\}$$

The level of charge of the batteries is expressed in terms of stored charge (Ah). The stored charge of the battery of the k^{th}

vehicle has an initial level of $s_k(0)$. The future stored charge of a battery is estimated by:

$$s_k(t) = s_k(t-1) + \lambda \; x_{k,t} \; \Delta t, \tag{1}$$

where $x_{k,t}$ is current supplied to the vehicle, Δt is the size of the discretized time interval and λ is an efficiency factor (we use 0.9) that takes into account energy lost due to AC/DC conversion and cooling.

B. Decision Variables and Objective

Control of vehicle charging rates can be expressed in terms of maximum allowed current, as for example in the J1772 standard [25]. The decision variables are therefore the currents supplied to all charging vehicles over all intervals in the charging horizon, which may be denoted by the matrix $\bar{x}$ having dimension $K \times T$. $\bar{x}$ can be rewritten as a vector x using its column vectors for notational convenience.

When user preferences are not taken into account, the objective is simply to allocate as much current to the vehicles as possible:

$$\max_x \sum_{t=0}^{T} \sum_{k=1}^{K} x_{k,t} \tag{2}$$

Note that we make a distinction between the current allocated to each vehicle as a result of this optimization, and the actual current drawn by each vehicle's battery charging control system. The charging control system may choose not to draw the full amount of current it is allocated due to battery-dependent constraints involving state-of-charge, state-of-health, temperature, efficiency of charging at varying rates, etc. This is a complex control decision in its own right and we do not pursue it in detail here, but focus instead on the maximum current that can be made available in the first place.

C. Constraints

The full set of system constraints may be written in the standard format $Ax \le b$, where the matrix A and the vector b result from the grid conditions. Again the reader is referred to [9] for full details. The constraints include the following:

1) Thermal Limits: Network components have nominal ratings that should not be exceeded. For the transformer:

$$V_{Tx} \; x_{\phi,t} \le \frac{1}{3} \; P_{Tx}^{\max} \times 130\%, \quad \phi \in \{\phi_1, \phi_2, \phi_3\}$$

where V_{Tx} is the phase-to-neutral voltage at transformer and $P_{Tx}^{\max}$ is the transformer's nominal power rating (in other words, maximum power is capped on each phase and a 30% excess is allowed, in line with typical operating philosophy). Similarly, there are current ratings for each phase of the backbone and for each service line:

$$x_{\phi,t} \le x_{\phi}^{\max}, \quad \phi \in \{\phi_1, \phi_2, \phi_3\}$$
$$x_{k,t}^{\text{tot}} \le x_k^{\max}, \quad \forall k \in \mathcal{K}$$

where $x_k^{\max}$ is the current rating of service line k.

2) Voltage Drop: Line impedance can lead to voltage at houses far from the transformer dropping below minimum required levels. As we have previously shown in [9], for networks that are mainly resistive (as is often the case at the distribution level, where power factor is usually high), it is possible to express the voltage at each house as a linear expression of the currents drawn by all other houses in the network. This linearization is an approximation to the true voltage at each house, but when power factor is high it turns out to be a sufficiently accurate representation. In our previous study, household voltages approximated in this way were shown to vary from true household voltage by 1% on average [9].

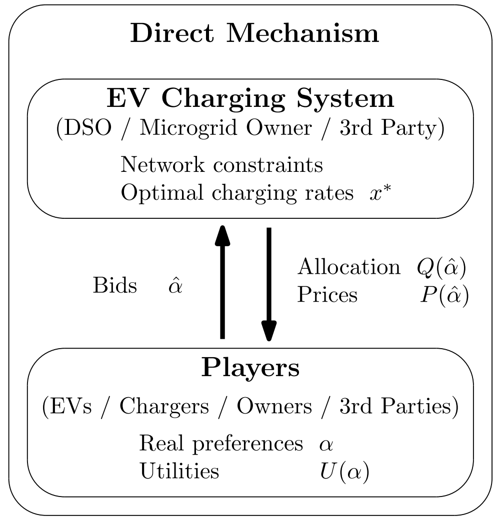


Fig. 1. Outline of the EV charging direct mechanism.

Voltage drop can therefore be expressed as an individual constraint for each house as

$$V_{Tx} - \sum \left[V_{a,b}^{\text{drop}} \right]_{h,t} > V^{\min}$$

where $[V_{a,b}^{\text{drop}}]_{h,t}$ are all piecewise voltage drops from the distribution transformer to house h at time t, and $V^{\min}$ is the minimum allowed voltage.

3) Phase Unbalance: An unbalanced system can have negative effects on electrical equipment and leads to higher current in the neutral, which in turn leads to increased losses. This is prevented using:

$$\frac{\left| x_{\phi,t} - \frac{1}{3} \sum_{\phi} x_{\phi,t} \right|}{\frac{1}{3} \sum_{\phi} x_{\phi,t}} < p, \quad \phi \in \{\phi_1, \phi_2, \phi_3\}$$

where p is the maximum allowed percentage unbalance.

III. Mechanism Design Framework

In this section the optimal charge allocation method described in Section II is extended to take into account individual user preferences using a market mechanism. An outline of the mechanism is presented in Fig. 1; each of the components of this mechanism are described in the following subsections.

A. Charging System, Players, Utilities, Preferences

The mechanism specifies the interactions between the EV charging system and the charging vehicles. The "EV Charging System" would be run by any entity that aims to manage demand in a way that is safe for the network – this could be the distribution network operator, an owner of a community grid or microgrid, or a third party that wants to avoid negative network impacts.

We define as the "player" any agent that is making decisions regarding the charging of an electric vehicle. This might be the vehicle itself, the vehicle's charging system, the vehicle owner, or a third party such as a demand response aggregator.

Each player aims to maximize the level of charge of their battery. However, it is natural to expect that players have different preferences regarding how urgently they wish to charge their vehicle. This concept is formalized by extending (1) and associating each player k with an individual utility:

$$U_k(x_{k,t}) := \alpha_k s_k(T) = \alpha_k \left(s_k(0) + \lambda \sum_{t=0}^{T} x_{k,t} \Delta t \right), \quad (3)$$

where the player-specific parameter $\alpha_k \in \mathbb{R}^+$ reflects the charging preference. This utility function quantifies the extent to which players value a final state of charge or are "willing to pay" for it. Representing the player preferences using a real variable captures the most general case. In practice players may be restricted to quantized values corresponding to e.g., tiered plans offered by demand aggregators.

There are a number of ways that such a utility function might be chosen, and it could be dependent on factors such as state of charge, desired departure time, remaining budget, etc. In addition, players might want to adjust their preferences over time in response to changes in the system state. For the purposes of this paper, however, the formulation in (3) is chosen for its computational and conceptual simplicity.

Consider a system that controls the vehicle charging process centrally by adjusting the charging currents x. Following the objective and constraints described in Section II, the global optimization problem that aims to maximize the aggregate utility of all players (also referred to as social welfare maximization [26]) is:

$$\max_x \sum_{t=0}^{T} \sum_{k=1}^{K} \alpha_k x_{k,t} \ , \quad \text{s.t.} \quad Ax \leq b \quad (4)$$

Note that the system constraints described in Section II impose an upper bound on the charging rates x. Hence, EV charging is a resource allocation problem where multiple decision makers with varying preferences share a limited resource (the current flowing to the batteries).

The optimization problem (4) has been addressed in [9] for the case when the real preferences of the players, as captured by α_k, are known and equal. However, the preferences are private information and may not be known by the system solving the global optimization problem and deciding on current allocations. Furthermore, there is a risk that players may attempt to cheat the system by misrepresenting their preferences such that the solution of (4) leads to an unfairly high charging rate for cheating players at the expense of honest ones.

A possible way of addressing this problem is adopting a classical market-based approach, where participants pay for the charging current (limited resource) according to their preferences. While some users may be ready to pay a large amount for faster battery charging (higher current) others may not value it as much. In a quasi-linear setting, money (or an equivalent measure such as internal system credits) acts as a metric which quantifies the participants' utilities or their willingness to pay.

B. Bids, Pricing Function, Allocation

We introduce the following notation, which will be used throughout the following sections. The elements of the vector $\alpha := [\alpha_1, \ldots, \alpha_K]$ in conjunction with the individual utility functions (3) represent the player preferences in the system at a given time, and are called "types" in the mechanism design literature [26]. α_k refers to the k^{th} element of vector α, while α_{-k} refers to all elements of α except the k^{th} (in line with standard notation from the game theory community). We also make a distinction between the "claimed" type, $\hat{\alpha}_k$, and the "real" type, α_k.

We focus here on direct mechanisms without loss of any generality due to the relevation principle in mechanism design [26]. In a *direct mechanism*, the participants play a game where they *bid* by communicating their claimed types, $\hat{\alpha}_k$, to the system. Ideally, the communicated types should match the real ones, $\hat{\alpha}_k = \alpha_k \ \forall k$. However, since some players may try to cheat for their own benefit, the participants' desire for charging their vehicles is balanced by a counteracting *pricing function* $P_k(\hat{\alpha})$. The pricing function communicates to participants the system constraints and overall demand; it is an important part of the mechanism and is carefully chosen later to ensure several key properties for the mechanism. For convenience of notation, let

$$x^* = \underset{x,\, Ax \leq\ b}{\arg\max} \ \sum_{t=0}^{T} \sum_{k=1}^{K} \alpha_k x_{k,t}$$

In other words, x^* is the vector of charging rates that optimally solves the global optimization problem in (4).

It should be emphasized here that since the global optimization problem takes network constraints into account (as specified in Section II-C) the vector x^* contains a set of charging rates over the horizon for each vehicle that already ensure that no network constraints will be violated.

We now define the *allocation function*:

$$Q_k(\hat{\alpha}) = s_k(0) + \sum_{t=0}^{T} \bigl(x^* \ \Delta t\bigr) \quad \forall k, \quad (5)$$

which corresponds to the optimal final states of charge $s_k(T)$ for each player k given the preference vector α. Given such an allocation function, each player k then aims to minimize their own cost J_k, defined as the difference between the pricing and utility functions:

$$J_k(\hat{\alpha}) = P_k(\hat{\alpha}) - \alpha_k Q_k(\hat{\alpha}) \quad (6)$$

Note that the actual individual player utility is a function of the real preference α_k whereas both the pricing and allocation functions calculated by the system have to rely on the claimed preference vector $\hat{\alpha}$.

The described interaction between the system and the players can be analyzed as a noncooperative game.

Definition 1 (EV Charging Game): The EV charging game $\mathcal{G} = \{\mathcal{K}, \hat{\alpha}_k \in \mathbb{R}^+, J_k\}$ is defined by the set of players $\mathcal{K}$ (representing EVs), their communicated preferences $\hat{\alpha}_k \in \mathbb{R}^+\ \forall k$, and cost functions $J_k\ \forall k$ defined in (6).

The **Nash equilibrium** (NE) is a widely-accepted and useful solution concept in strategic games. At the NE, no player has an incentive to deviate from it while others play according to their NE strategies. The NE $\hat{\alpha}^*$ of the game $\mathcal{G}$ is formally defined as

$$\hat{\alpha}_k^* := \arg\min_{\hat{\alpha}_k} J_k\big(\hat{\alpha}_k, \hat{\alpha}_{-k}^*\big),$$

where $\hat{\alpha}_{-k}^* = [\hat{\alpha}_1^*, \ldots, \hat{\alpha}_{k-1}^*, \hat{\alpha}_{k+1}^*, \ldots, \hat{\alpha}_K^*]$. The NE is at the same time the intersection point of players' best responses which can be obtained by solving (6) individually.

Proposition 1: If the player cost functions $J_k,\ k \in \mathcal{K}$ defined in (6) are quasi-convex in their arguments, then the EV charging game $\mathcal{G} = \{\mathcal{K}, \hat{\alpha}_k \in \mathbb{R}^+, J_k\}$ admits a Nash equilibrium solution, $\hat{\alpha}^*$.

Proof: In the K-player strategic (noncooperative) static game $\mathcal{G} = \{\mathcal{K}, \hat{\alpha}_k \in \mathbb{R}^+, J_k\}$ the strategy space $\mathcal{X} \in [\mathcal{R}^+]^k$ is convex, compact, and non-empty. The player cost functions $J_k,\ k \in \mathcal{K}$ defined in (6) are continuous on $\mathcal{X}$ and quasi-convex in their arguments $\hat{\alpha}_k$. Thus, from Theorem 12.3 in [27], the game $\mathcal{G}$ admits a Nash equilibrium solution, $\hat{\alpha}^*$, in pure strategies. ■

A stronger equilibrium concept is the **Dominant Strategy Equilibrium** (DSE), which is defined as

$$\hat{\alpha}_k^D := \arg\min_{\hat{\alpha}_k} J_k(\hat{\alpha}_k, \hat{\alpha}_{-k}),\ \forall \hat{\alpha}_{-k}\, \forall k,$$

i.e., the players choose the dominant strategy regardless of the actions of others.

The following definitions describe various properties of a mechanism and its corresponding game counterpart:

Definition 2 (Efficiency): A mechanism is said to be efficient if its outcome, i.e., the NE or DSE of the corresponding strategic game, x^*, satisfies a given global objective such as the one in (4).

Definition 3 (Strategy-proof): A mechanism is said to be strategy-proof, if and only if, the corresponding game admits a DSE that reveals the true user types (preferences).

Definition 4 (Revelation): In a strategy-proof mechanism, each rational user acts according to its own true utility or reveals its own true type regardless of the actions of others, i.e., does not try to mislead the designer.

C. Direct EV Charging Mechanism

A direct mechanism is presented now where players bid their preferences, $\hat{\alpha}$, to the system to receive an allocation of $Q_k(\hat{\alpha})$ at the end of their declared charging periods and accordingly pay $P_k(\hat{\alpha})$. Devising an incentive-compatible direct mechanism first is appropriate since the well-known *revelation principle* states that if no such mechanism exists for this case then it is not possible to develop any indirect mechanism [26].

The allocation function (5) ensures that the intended direct mechanism is efficient. By carefully choosing the pricing function, $P_k(\hat{\alpha})$, the mechanism is ensured to be strategy-proof. Thus, the following direct EV Charging Mechanism, $\mathcal{M}_{EV}$, is obtained.

Theorem 1: The direct EV Charging mechanism

$$\mathcal{M}_{EV} =< \mathcal{K}, U_k, \hat{U}_k, Q_k(\hat{\alpha}), P_k(\hat{\alpha}) >,$$

defined by

1) the set of players $\mathcal{K}$ acting on behalf of the EVs, and the vector α representing their true utilities $U_k = \alpha_k s_k(T)$,
2) the player bids $\hat{\alpha}_k\ \forall k \in \mathcal{K}$, which reflect their claimed utilities, $\hat{U}_k = \hat{\alpha}_k s_k(T)$.
3) the allocation function $Q_k(\hat{\alpha})$ defined in (5),
4) the marginal pricing function
$$\frac{\partial P_k(\hat{\alpha})}{\partial \hat{\alpha}_k} = \hat{\alpha}_k \frac{\partial Q_k(\hat{\alpha})}{\partial \hat{\alpha}_k} \tag{7}$$
and the pricing function (which directly follows)
$$P_k(\hat{\alpha}) = \hat{\alpha}_k Q_k(\hat{\alpha}) - \int_0^{\hat{\alpha}_k} Q_k(\varphi, \hat{\alpha}_{-k}) d\varphi. \tag{8}$$

is both efficient and strategy-proof.

Outline of the Proof: We first show that $\partial Q_k(\hat{\alpha})/\partial \hat{\alpha}_k > 0$ by contradiction. Assume otherwise. Then, a player can choose a smaller $\hat{\alpha}$ without decreasing its allocation which gives that player a lower weight in the global optimization problem (4). This contradicts the fact that players with lower weights receive less allocation.

The derivative of (6) with respect to $\hat{\alpha}_k$ yields

$$\frac{\partial J_k(\hat{\alpha})}{\partial \hat{\alpha}_k} = \frac{\partial P_k(\hat{\alpha})}{\partial \hat{\alpha}_k} - \frac{\partial\big(\alpha_k Q_k(\hat{\alpha})\big)}{\partial \hat{\alpha}_k}$$

Using (7), this becomes

$$\frac{\partial J_k(\hat{\alpha})}{\partial \hat{\alpha}_k} = \hat{\alpha}_k \frac{\partial Q_k(\hat{\alpha})}{\partial \hat{\alpha}_k} - \left(\alpha_k \frac{\partial Q_k(\hat{\alpha})}{\partial \hat{\alpha}_k} + \frac{\partial \alpha_k}{\partial \hat{\alpha}_k} Q_k(\hat{\alpha})\right)$$

And therefore,

$$\frac{\partial J_k(\hat{\alpha})}{\partial \hat{\alpha}_k} = (\hat{\alpha}_k - \alpha_k) \frac{\partial Q_k(\hat{\alpha})}{\partial \hat{\alpha}_k}.$$

Since $\partial Q_k(\hat{\alpha})/\partial \hat{\alpha}_k > 0$,

$$\frac{\partial J_k(\hat{\alpha})}{\partial \hat{\alpha}_k}(\hat{\alpha}_k - \alpha_k) = \big(\hat{\alpha}_k - \alpha_k\big)^2 \frac{\partial Q_k(\hat{\alpha})}{\partial \hat{\alpha}_k} \geq 0,$$

with equality at $\hat{\alpha}_k = \alpha_k$. Given the fact that the derivative term is monotonic on either side of α_k, $J_k(\hat{\alpha})$ achieves a global minimum at α_k.

As this holds for all players, $\hat{\alpha} = \alpha$ is the DSE of the EV charging game $\mathcal{G}$ in Definition 1 and the mechanism is strategy proof. Furthermore, the allocation $Q_k(\hat{\alpha})$ in (5) solves the global optimization problem (4) with true player preferences. Hence, the mechanism is efficient. ■

Note that the mechanism $\mathcal{M}_{EV}$, being both efficient and strategy-proof, belongs to the class of Groves Mechanisms which includes the famous Vickrey-Clarke-Groves (VCG) mechanism [26]. The developed mechanism specifically differs from VCG in its pricing function (payment rule) as well as in explicitly taking into account the distribution network constraints.

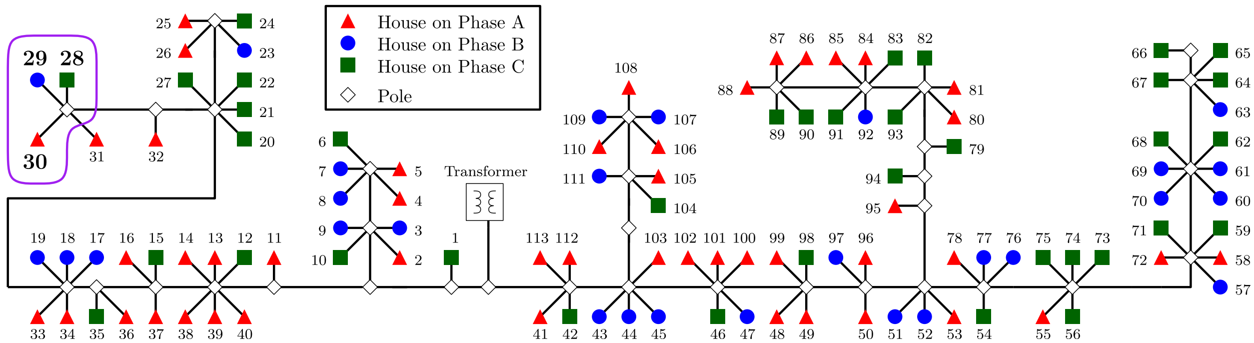


Fig. 2. Diagram of a real network. Voltages at houses 28, 29, and 30 (as outlined) are explored in Table I.

D. *Implementation, Convergence, Communication*

The actual implementation of the direct mechanism described in Section III-C proceeds as follows: in each interval, all players submit their bids $\hat{\alpha}_k$ to the system. The system collects these bids and solves the optimization problem in (4), which provides the charging schedules x^*. The system calculates final states of charge Q_k (5) and the prices P_k (8) and passes these on to the EVs, and allocates charging according to the calculated schedule. In the next iteration, EVs can update their bids based on their preferences, and the mechanism is repeated.

The number and frequency of these iterations depends on two separate time scales.

Over the longer term, the full charging schedule resulting from this mechanism clearly needs to be recalculated at regular intervals, to take into account changing conditions such as vehicle arrival and departure, as well as changes in underlying household demand. Such recalculation can be conducted in an event-based manner (e.g., every time a vehicle arrives) or in a time-based manner (e.g., every 15 minutes).

Over the shorter term, this iterative process could provide an opportunity for players to adjust their preferences in response to the allocation they receive. In other words, every time a full charging schedule is calculated, there could be multiple passes of back-and-forth communication between the EV Charging System (the central coordinating entity) and the players. Individual players could "bid in" at a low preference, but increase this preference in response to a less-than-desirable allocation (or to achieve a target charge level), for example. There are many ways such a repetitive auction could be implemented, and we leave this as future work. In the simulation results presented in this paper we consider only a single iteration each time the full charging schedule is calculated.

The communication requirements for this method involve the transfer of k values (players to system) and $|Q_k| + |P_k| = 2k$ values (system to players) for each iteration. For a repeated auction scenario the communication requirements could therefore grow quickly and require robust communication infrastructure. For the single auction method presented in this paper, however, the communication requirements are not that extensive and could be implemented using a range of standard methods such as smart meters or mobile messaging.

TABLE I
VOLTAGES AT HOUSES 28, 29, AND 30 IN THE NETWORK SHOWN IN FIG. 2

House number	28	29	30
Equivalent loads at every house	228.3	238.6	217.9
Adding a vehicle at house 30 only	228.1	239.8	**215.1**
Adding a vehicle at house 28 only	225.4	238.4	219.1
Adding vehicles at houses 28 and 30	225.2	239.5	216.4

IV. CREATING A FAIR MARKET

A key consideration when introducing user preferences is the notion of *fair market access* – i.e., the concept that all market participants should have an equal opportunity to participate in the market, without any individual having an unfair advantage from the outset. When that market is based on EV charging and network constraints are taken into account, there is a risk that access to the available resources that the market is based on (in this case charging capacity) is unfairly distributed.

In unbalanced three-phase networks, the specific locations of loads will define the extent of their impact on system stability. Different phase loading levels can lead to neutral point shift and current in the neutral line. These differences can have an impact on charging rates when optimizing for the system as a whole as in (4). Vehicles having less of a negative impact on the network will be charged faster than vehicles having more of a negative impact (as also described in [2], where a modified objective is introduced to increase fairness).

To demonstrate these effects, an example is provided in Figure 2. This diagram presents a model of a real network in Melbourne, Australia in which all phase connections and line specifications are true to the real network. The houses are allocated to phases A:B:C at a ratio of 48:28:37. When every house is assigned a load of 2.2kW at power factor 0.9, the voltages at the houses labeled 28, 29, and 30 are as shown in the first row of Table I. As can be seen, despite their equivalent distances to the transformer, unbalance in the network leads to significant voltage differences across the phases.

When an electric vehicle (3.45kW at power factor 1.0) is added to house 30, the voltages are as shown in the second row. With local distribution code requiring a minimum voltage of 216V, this house would have a voltage below minimum required levels. However, when an electric vehicle is added instead to house 28, the voltages are as shown in the third row, and all voltages remain within required limits. Similar differences can be demonstrated when comparing vehicles on the same phase, but at different distances to the transformer. The objective in (4) can therefore introduce significant bias into the way that vehicle charging is allocated.

This problem is compounded by an additional effect. The bottom row in Table I shows the voltages when electric vehicles are added to *both* houses 28 and 30. Despite the increased total load on the network, the voltage at house 30 is once again within acceptable limits. The additional load at house 28 on phase C *rebalances* the network, and thereby reduces voltage drop on the most heavily loaded phase A, enabling additional charging capacity. This example therefore illustrates an important point: couplings between individual users in a three-phase distribution network play an important role and cannot be ignored.

If both user preferences and network constraints are therefore to be taken into account at the same time, a level playing field must be created that allows all users equal access to the available resources. Two players submitting the same bid should each be allocated the same charging current. This can be achieved by introducing an additional set of "fairness constraints":

$$\sum_{t=0}^{T} x_{i,t} = \frac{\hat{\alpha}_i}{\hat{\alpha}_j} \sum_{t=0}^{T} x_{j,t} \quad \forall i, j \in \mathcal{K}, \;\; i \neq j \tag{9}$$

Using the above constraints, players' allocations will proportionally reflect their bids. In an alternative formulation, it would be possible to adjust this proportionality as desired, depending on what levels and types of fairness are required – this is discussed in more detail in Section VI.

V. Simulation Results

A. Simulator, Model, Data, Preferences

To demonstrate the mechanism proposed in this paper we conducted a series of simulations using our simulation environment,[1] as previously described in [9]. This consists of a C++ wrapper that uses MATLAB SimPowerSystems for model building and load flow analyses.

The network model used throughout these simulations is the same one previously shown in Fig. 2. As already specified in Section IV, this model is based on a real network in Melbourne, Australia in which all phase connections and line specifications are true to the real network. For each backbone and for each service line the real segment length and impedance per unit length values, as provided by the network operator, were used.

Household demand is simulated using data measured at the distribution transformer of this network. Each house is assigned the same average demand profile. This is not a fully realistic way to simulate household demand, but it provides a common base across all households that makes it easier to assess the relative impacts of the electric vehicles.

[1]POSSIM: POwer Systems SIMulator, available at http://www.possim.org

TABLE II
Vehicles and Preference Assignment

#	α	#	α	#	α	#	α	#	α
2	2	18	2	46	1	74	2	96	2
3	1	21	1	59	2	78	1	102	1
9	3	31	2	60	1	84	3	113	1
12	1	42	1	63	2	88	1		
14	3	43	2	72	1	93	3		

Vehicles are assigned to the houses as shown in the left columns of Table II. In total there are 23 vehicles, for an uptake of 20%, distributed across the phases at a ratio of 10:6:7 (reflecting roughly the same ratio of houses to phases). All vehicles are simulated to arrive at home at 6pm with a state of charge of 20%. Again, this is not a realistic scenario, but it is used to enable a clear demonstration of the different behaviors of different algorithms compared in this section. There is no reason the market mechanism proposed in this paper could not be applied to scenarios where houses have vastly different demand profiles and vehicles arrive and depart at different times (with different states of charge); we only use these simplified scenarios here for easier demonstration of the mechanism's properties.

All simulations use fully complex, three-phase, unbalanced load flow analyses, as conducted in MATLAB SimPowerSystems, to determine total network load, as well as voltages and currents at all houses and throughout the system.

B. Uncontrolled Charging

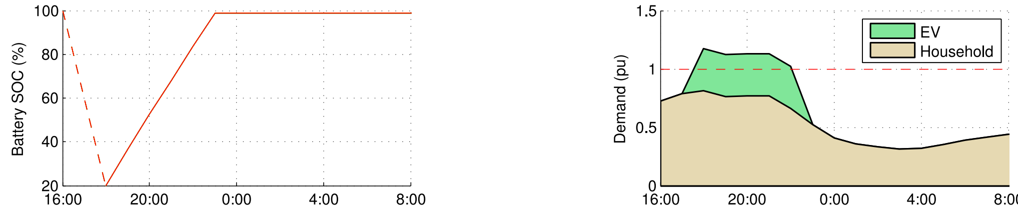
For purposes of comparison, the first set of results present the **Uncontrolled** case. In this scenario, vehicles are charged at their maximum rates until a full state of charge is reached. There is no effort to respect network constraints and no individual preferences are taken into account. The results are presented in Fig. 3. All vehicles charge at the maximum possible rate of 3.45kW (Fig. 3a). This leads to overload of the transformer (Fig. 3b) and voltage drops below the minimum threshold at several houses. In other words, under uncontrolled charging this network would not be able to sustain an electric vehicle uptake of 20%.

C. Market-Based Charging

The next sets of results demonstrate the **Market-Based Charging** arising from the direct mechanism detailed in Section III with the objective (4).

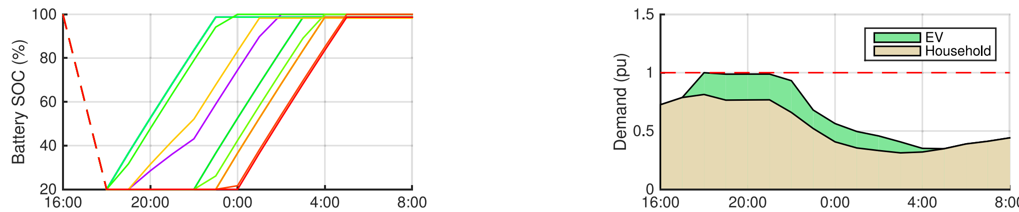
When all players have the same preference and fairness is not enforced, the results are as shown in Fig. 4. Different vehicles are allocated different rates of charge, resulting from their differing impacts on the network as a result of their respective locations (Fig. 4a). All vehicles are charged by 5am, and total demand (Fig. 4b) as well as voltages at all houses remain within required limits due to the network constraints. In other words, with market-based charging this network would be able to sustain an electric vehicle uptake of 20%, and all vehicles would be charged in time.

When players are randomly assigned one of three different preferences (as per Table II), the results are as shown in Fig. 5. Some vehicles are now preferred over others, due to



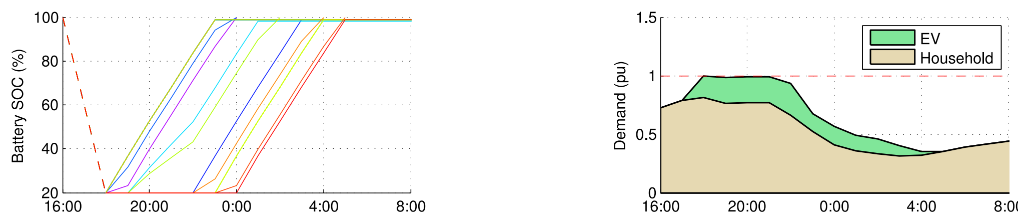
(a) Vehicle charging rates (b) Total demand

Fig. 3. Uncontrolled charging.



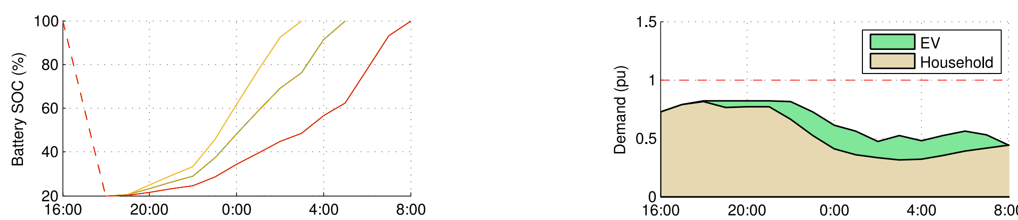
(a) Vehicle charging rates (b) Total demand

Fig. 4. Market-based charging with network constraints, preferences equal, no fairness constraints.



(a) Vehicle charging rates (b) Total demand

Fig. 5. Market-based charging with network constraints, preferences different, no fairness constraints.



(a) Vehicle charging rates (b) Total demand

Fig. 6. Market-based charging with network constraints, preferences different, with fairness constraints.

their higher preferences, but the different charging rates do not accurately reflect players' relative preferences since some vehicles remain highly favored over others as a result of their locations in the network (Fig. 5a). Network constraints continue to be respected and total demand stays within required limits (Fig. 5b).

When in addition the fairness constraints (as detailed in Section IV) are introduced, the results are as presented in Fig. 6. The vehicles are clearly grouped into three separate levels of charging, as defined by their preferences (Fig. 6a). Total demand (Fig. 6b) and voltages remain within required limits. Vehicles having the same preference receive the same levels of charge.

However, vehicles having the lowest preference are not fully charged until 8am. The cost of "leveling the playing field" is therefore shown to be significant: while individual users' preferences relative to one another are respected in this mechanism, the total system throughput is significantly reduced. In fact, the best charging profile in the fairness-constrained market-based method is only slightly better than the worst charging profile when fairness is ignored. This is emphasized in Fig. 7, which clearly shows the reduction in total energy supplied to the vehicles when a level playing field is enforced.

The prices paid by users in the market-based method are shown in Fig. 8. As can be seen, three distinct levels of prices emerge that reflect the three different preference levels adopted

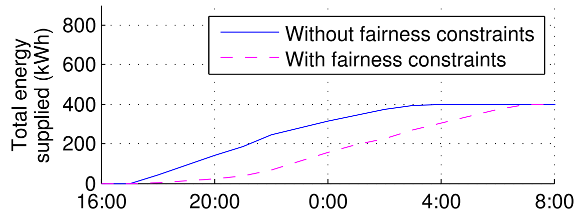


Fig. 7. Impact of fairness constraints.

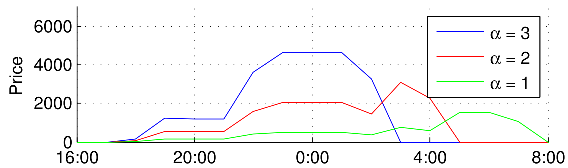


Fig. 8. Three tiers of prices paid by users in the market-based charging method.

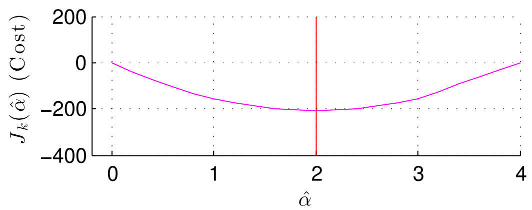


Fig. 9. Demonstrating that the mechanism is strategy-proof.

by the vehicles. When the vehicles having the highest preference are fully charged, only two pricing levels remain. When the vehicles having the middle preference are fully charged only a single price level remains.

D. Demonstrating the mechanism's properties

It follows naturally that the mechanism is efficient (as per Definition 2), since the outcome for each game is exactly the solution to the global objective in (4). In the next set of results it is demonstrated that the mechanism is also strategy-proof.

To demonstrate this, a wide range of possible values for $\hat{\alpha}_{96}$ were examined. In other words, would it be possible for vehicle 96, having a true $\alpha_{96} = 2$ to benefit by representing its $\hat{\alpha}_{96}$ as having a different value? The results for one point in time (11pm) are shown in Fig. 9. The x-axis present the range of claimed preferences, with the curve showing the true cost $J_{96}(\hat{\alpha})$. It can be seen that there is a minimum at $\hat{\alpha} = 2$, which is indeed the true preference α_{96}. Therefore the best strategy for vehicle 96 is to bid its true type.

VI. Discussion

The simulation results in Section V confirm the importance of considering network constraints when developing market-based methods for demand response. Due to geographic location and underlying network characteristics, some market players are likely to have significantly greater access to the available resources from the outset than others. The fairness constraints proposed in Section IV help towards alleviating this imbalance by enforcing access to the market that is proportional to players' preferences.

However, there are obvious drawbacks to such an approach. As illustrated in Figure 7, enforcing fairness leads to a significant reduction in total system throughput. In any such systems, there are inherent trade-offs involved in maximizing the use of available capacity, versus ensuring individual users receive their fair share according to what they are willing to pay. Even after the charging schedules are decided in our approach, there is remaining capacity available for further charging of some vehicles. How this should be allocated (if at all) remains an open question that depends on the needs and priorities that arise in the system under consideration.

It could, for example, be possible to allocate remaining capacity in such a way that all available network capacity is maximally utilized – as part of a two-stage optimization, in which stage one ensures fairness, and stage two ensures maximal use of resources. This could mean however that two players having the same preference could receive vastly different charging rates – a situation that could lead to customer dissatisfaction in the real world. We intend to explore alternative solutions in future work.

VII. Conclusion

A market-based direct mechanism for electric vehicle demand response was proposed that allows for individual user preferences to be applied. The mechanism is built on top of an existing optimal charging solution that takes into account the constraints in the distribution network. These constraints have important impacts on users' access to the available charging power, since some locations in the network will have a much stronger impact on network stability than others. To allow all users fair access to the market, "fairness constraints" are introduced that ensure that any two users submitting the same bid will also be allocated the same share of the resource. However, the fairness constraints significantly reduce the total throughput (i.e., the total energy supplied to the vehicles) that can be achieved.

The mechanism is shown to be efficient and strategy-proof, so it follows that users in this mechanism can not cheat the system by misrepresenting their preferences; the cost for each user accurately reflects that user's true preference. It has therefore been shown that individual user preferences can be incorporated into demand response for electric vehicles while still taking network constraints into account. This is of great value to demand response aggregators who may wish to offer different tiers of service to their users, while still ensuring that there are no negative impacts from EV charging on the underlying networks.

In future work we intend to examine further some of the issues around fairness, explore multi-stage auctions, and evaluate how distributed generation may affect these results.

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