

# Analysis of Constraints for Optimal Electric Vehicle Charging<sup>\*</sup>

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**Abstract:** The increasing uptake of electric vehicles is likely to put a significant demand on the electricity grid. However, expensive infrastructure upgrades can be avoided if some of the vehicle charging can be shifted to off-peak times. We express electric vehicle charging as a receding horizon optimisation problem in which inherent network limitations are taken into account as linear constraints. By tuning these constraints, it is possible to push total system performance in one direction or another as required, and to adapt the solution to a variety of networks and to different sets of network limitations. We explore in detail this tuning process and discuss the trade-offs involved. There are several emergent benefits from examining the constraints in this way: a capacity constraint can be used to prevent peak load increases; a phase unbalance constraint can be used to enforce system rebalancing; and a voltage limit constraint can ensure that vehicle charging does not push voltages outside allowed limits. Our conclusions are demonstrated in simulation studies that use a model of a real network, as well as real demand and vehicle travel data.

*Keywords:* Electric vehicles, receding horizon, optimisation, centralised charging, power systems

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## 1. INTRODUCTION

Electric vehicles are being promoted by many governments as a way to reduce greenhouse gas emissions in the personal transport sector, and their market share is steadily increasing. However, the charging of electric vehicles places a major demand on the electricity grid. There is a risk that vehicle charging will coincide with peak demand and lead to problems such as thermal overload, voltage instability, and phase unbalance (Kelly et al., 2009; Gerkensmeyer et al., 2010; Lopes et al., 2011). This problem may be alleviated by smart charging: shifting electric vehicle demand to off-peak times, such as overnight. A variety of approaches have already been proposed to allow this.

One set of approaches uses a decentralised methodology: vehicles (or vehicle chargers) make decisions on when to charge individually, without access to any knowledge of system state (Ma et al., 2010; Gan et al., 2011). Local measurements such as voltage can be used to estimate existing network load (Ganu et al., 2012; de Hoog et al., 2013). On the one hand, these methods do not require installation of costly metering and communication infrastructure; on the other, they may be difficult to regulate and may struggle to find the best solution.

A completely different set of approaches solves smart charging in a centralised manner: the timing and rate of charging for all vehicles is determined by a central solver with access to full system state (Clement-Nyns et al., 2010; Richardson et al., 2012; Zhan et al., 2012). Key measurements throughout the system can be used to guide this decision. The required communication and control infrastructure needs to be installed,

but a recent demonstration in the Australian Victoria Electric Vehicle Trial has demonstrated the feasibility of central load control where advanced metering is present (Z. Angelovski and K. Handberg, May 2013). A central solution can arguably find the best possible way of distributing available network resources among charging vehicles.

In previous work, we have proposed a way to express electric vehicle charging at the distribution level as a receding horizon optimisation problem that is linear in both the decision variables and the constraints (de Hoog et al., 2014). Using this formulation it is possible to achieve high levels of electric vehicle penetration without any adverse impacts on existing grid assets. However, key to the performance of this linear solution is the expression of the constraints: depending on the choice of parameters, system behaviour can be strongly pushed in one direction or another.

The work presented here distinguishes itself from our previous work by exploring in greater detail the impacts of individual constraints on the solution of the optimisation, and the trade-offs involved. The constraints we explore are related to inherent limitations in the low voltage electricity distribution network, and aim to:

- Prevent thermal overload of any network assets, such as lines and transformers
- Prevent excessive phase unbalance
- Prevent voltage at any point of connection in the network from falling outside of required limits

Finally, we also examine how the choice of charging horizon affects charging decisions.

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<sup>\*</sup> This work has been funded by a Linkage Grant supported by the Australian Research Council, Better Place Australia, and Senergy Australia.

## 2. ELECTRIC VEHICLE CHARGING AS A RECEDING HORIZON OPTIMISATION PROBLEM

We have previously expressed electric vehicle charging as an optimisation problem in detail (de Hoog et al., 2014); here we provide only a brief overview of our problem formulation.

### 2.1 Decision Variables

The goal of an electric vehicle charging system is to determine charge rates for all connected vehicles. While charge rates are typically expressed in terms of power (kW), we choose as our decision variables the currents (A) supplied to each vehicle. This is in line with the J1772 standard (SAE, 2001), and allows us to keep our problem formulation linear. We consider it a reasonable simplification since voltage is highly regulated and unlikely to vary much beyond 5% of nominal voltage.

Charging a vehicle can take several hours, so we consider the full charging horizon, making decisions on all vehicles' charge rates for every discrete interval within this horizon. Our decision variables are therefore the currents provided to each vehicle for charging,  $x_{k,t}$ , where  $k \in [1, 2, \dots, K]$  is the set of all vehicles that are connected and charging, and  $t \in [1, 2, \dots, T]$  is our charging horizon, divided into discrete intervals – a total of  $KT$  decision variables. Since vehicles may arrive or depart within the charging horizon, the full solution must be recalculated at every interval (say, every 5 minutes).

### 2.2 Objective Function

The objective may be tuned in one way or another depending on what goals are to be fulfilled. If we want to supply as much energy to the vehicle batteries as the network will allow, a simple objective is:

$$\max \sum_{k=1}^K \sum_{t=1}^T x_{k,t} \quad (1)$$

Alternatively, if we are concerned with fluctuations in the price of electricity that may affect the cost of charging, a price factor can also be included, which shifts charging to times when the price is low. In this case we want to minimise our objective:

$$\min \sum_{k=1}^K \sum_{t=1}^T p(t)x_{k,t} \quad (2)$$

where  $p(t)$  is a dynamically changing cost per unit of electricity. In such a scenario, it is essential to include charge targets for all vehicles, to ensure that not all  $x_{k,t}$  are zero. This can be done in a straightforward way via an additional constraint (for example, the sum of charging rates for each vehicle must reach state of charge of 90% within 6 hours).

### 2.3 Constraints

The constraints for our optimisation problem are defined by limitations in the low voltage network. We assume a 3-phase wye-connected network.

**Transformer:** It is important not to exceed the ratings of network assets. Distribution transformer loading may be limited on each phase of the network by the following constraints:

$$V_{Tx} x_{\phi,t} \leq \frac{1}{3} P_{Tx}^{\max} \quad \phi \in \{A, B, C\} \quad (3)$$

where  $V_{Tx}$  is output voltage of the transformer,  $x_{\phi,t}$  is total current on phase  $\phi$  at time  $t$  (including current due to both household and vehicle loads), and  $P_{Tx}^{\max}$  is the transformer's nominal power rating.

**Lines:** Line loading can be expressed as a constraint on a phase by phase basis by:

$$x_{\phi,t} \leq x_{\phi}^{\max} \quad \phi \in \{A, B, C\} \quad (4)$$

where  $x_{\phi,t}$  is again total current (household and vehicle) on phase  $\phi$  at time  $t$ , and  $x_{\phi}^{\max}$  is a maximum current rating of the line serving phase  $\phi$  of the network. Note that either (3) or (4) will be a tighter constraint than the other (making the other redundant), but it is still important to keep both in our problem formulation since transformer and line specifications can vary significantly from one network to the next.

**Phase Unbalance:** It is important for the system not to become too unbalanced, as this may have adverse effects on system components and lead to high neutral line currents. Unbalance is usually measured in terms of percent negative sequence voltage ( $|V_-| / |V_+|$ ), but this is difficult to linearise. Instead we limit unbalance using:

$$\frac{|x_{\phi,t} - \frac{1}{3}x_t^{\text{total}}|}{\frac{1}{3}x_t^{\text{total}}} < q\% \quad \phi \in \{A, B, C\} \quad (5)$$

where  $x_t^{\text{total}}$  is the total current in the system:

$$x_t^{\text{total}} = \sum_{\phi \in \{A, B, C\}} x_{\phi,t}$$

In other words, no single phase load may exceed average phase load by more than  $q\%$ .

**Voltage Drop:** Voltage at every point of connection in the distribution network must be maintained within upper and lower limits, according to local distribution codes<sup>1</sup>. Voltage dropping below this limit can lead to reduced lifetimes of appliances and other loads. This can occur when vehicles draw high levels of current which, due to line impedance, incur losses between the distribution transformer and the house. We approximate voltage drop in a linear way by assuming that distribution networks are mainly resistive (a common assumption in low voltage network modelling), and by examining the individual voltage drops in each piecewise segment of the line between the transformer and house  $k$ :

$$V_{Tx} - \sum [V_{a,b}^{\text{drop}}]_k > V^{\min} \quad (6)$$

where  $V_{Tx}$  is the source voltage at transformer,  $V^{\min}$  is the minimal allowed voltage, and  $[V_{a,b}^{\text{drop}}]_k$  are all piecewise voltage drops from source to house  $k$ . In other words, each house will have its own unique constraint that is still linear in terms of the currents through all other houses (and vehicles) in the network. (For a more detailed description of how we model voltage drop, refer to de Hoog et al. (2014)).

We have thus expressed electric vehicle charging as a receding horizon optimisation problem. The solution to this problem may be recomputed at each discrete interval, allowing for dynamic vehicle arrival and departure to be taken into account. The rest of this paper examines the impact of these constraints on system behaviour.

<sup>1</sup> In Australia, voltage must be within +10% / -6% of nominal voltage 230V, in other words within [216V 253V].

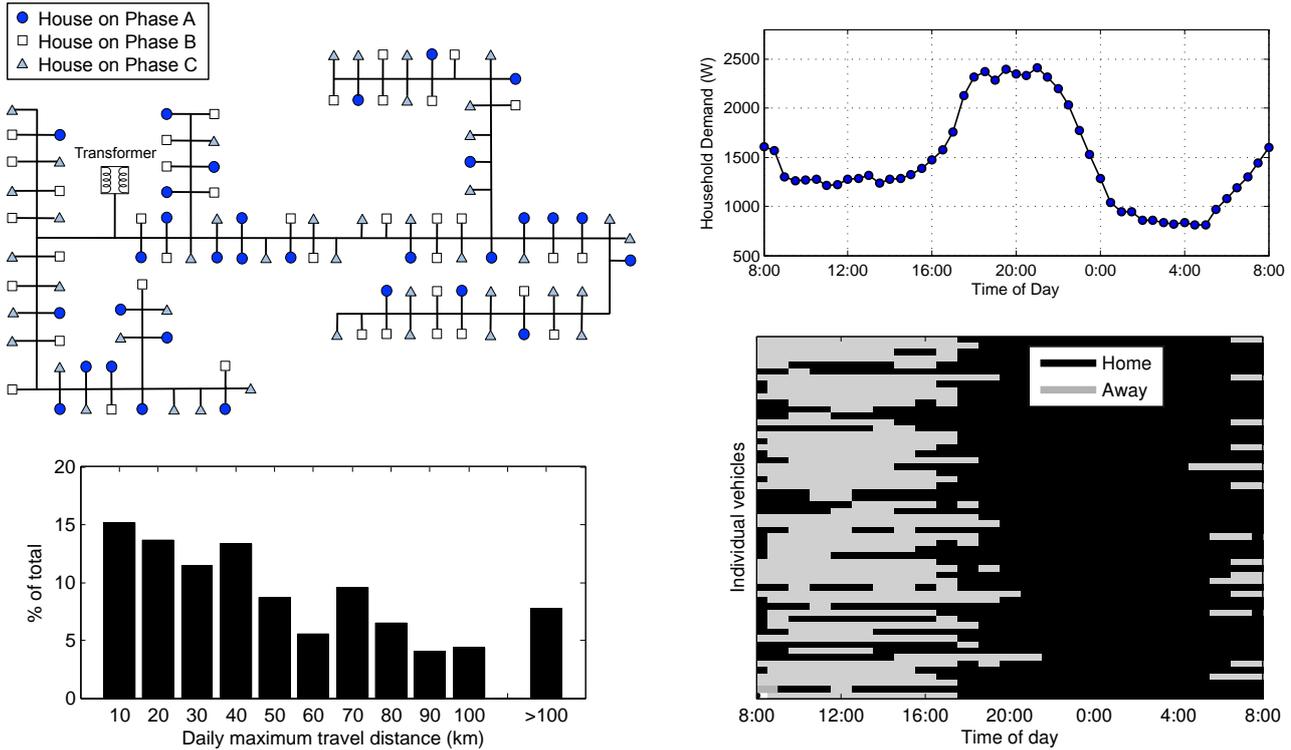


Fig. 1. Network model and data used for case study.

**Top left:** A diagram of the distribution network model used, based on a real network in northern Melbourne, Australia.

**Top right:** Average household demand for a weekday in June 2012, as measured at the transformer of this network.

**Bottom right:** Vehicle travel profiles for 57 vehicles obtained in the local government area that this network is located in.

**Bottom left:** Histogram of daily travel distances for the vehicle travel records obtained in this local government area.

### 3. SIMULATION FRAMEWORK

To implement our linear program and test our constraints, we ran a series of simulations using POSSIM<sup>2</sup>, which uses a MATLAB SimPowerSystems backend for model building and load flow analysis. We use a model of a real neighbourhood in northern Melbourne having 112 houses served by a 300kVA transformer (Fig. 1, top left). Our model is fairly unbalanced, with 31 houses on phase A, 38 houses on phase B, and 43 houses on phase C. To model household demand we use data provided by the network operator that was obtained in June 2012, a time of relatively high demand (Fig. 1, top right).

To model vehicle demand, we use data obtained by a state-wide travel survey (Victorian Integrated Survey of Travel and Activity, 2009). The resolution of this dataset allows us to choose vehicle travel profiles specific to the Local Government Area that our network is located in, which means that we have both realistic travel distances (to know charging demand) and timing of vehicle arrivals and departures (to know when they are available for charging). A histogram of typical travel distances is presented in Fig. 1 (bottom left), and the average daily vehicle travel distance in this neighbourhood is 44.6km. The vehicle travel profiles we use in our case studies are displayed in Fig. 1 (bottom right); clearly there is a lot of flexibility for scheduling charging overnight.

### 4. RESULTS

As a case study, we examined a scenario in which there is an uptake of 50%, in other words there are electric vehicles at half of the households in this network. While such high levels of electric vehicle ownership are unlikely to be reached for some time, we considered it a worthwhile exercise to examine the potential impacts on the network and to better demonstrate the importance of constraint choice.

#### 4.1 Uncontrolled charging

In the first set of simulations, we allowed all vehicles to charge at their maximum possible rates (3.45kW at a standard 230V 15A outlet) whenever they arrived at home, and allowed them to charge to completion. The results are presented in Fig. 2.

Fig. 2a presents total demand. Most vehicle charging occurs at peak demand time, leading to a 35% increase in peak demand. Individual voltages at points of connection of all houses on phase C (the most heavily loaded phase) are shown in Fig. 2b – each line represents voltage at one house. Notably, during peak times voltages at 20 houses drop below the minimum threshold of 216V due to additional vehicle demand. Phase unbalance (in terms percent negative sequence voltage,  $|V_-| / |V_+|$ ) is shown in Fig. 2c. A peak of 4.28% is reached. Fig. 2d presents the state of charge of three vehicles' batteries, with each line representing one vehicle. Dashed lines indicate that a vehicle

<sup>2</sup> POSSIM: POver Systems SIMulator, available at www.possim.org

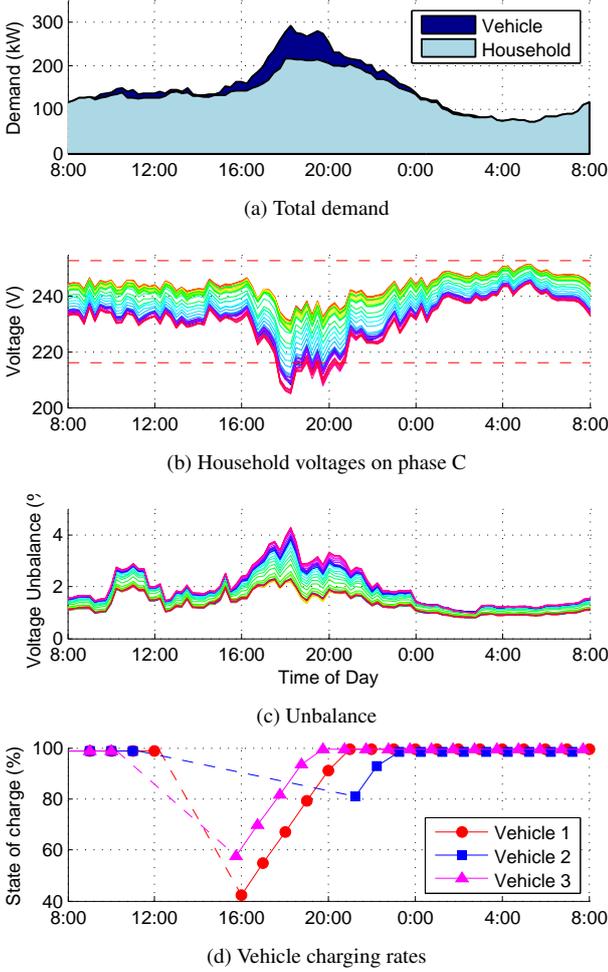


Fig. 2. Uncontrolled charging: all vehicles charge at the maximum possible rate as soon as they arrive at home, and charge to completion. There is a significant increase in peak demand, voltages at many houses drop below minimum required levels, and a high level of voltage unbalance is reached. However, vehicles charge quickly.

is not at home. As can be seen, all vehicles charge at constant rates and reach full charge as quickly as possible.

This network would not be able to sustain a 50% penetration of electric vehicles under uncontrolled charging conditions, due to low voltage at parts of the network distant from the transformer.

#### 4.2 Optimal charging

In our next set of simulations, we applied the objective detailed in Equation (1) and the constraints described in Section 2.3. For this set of results (Fig. 3) we limited demand at the transformer to 220kW and voltage at each house to 216V.

Peak demand hardly increases (due to our 220kW cap), and much of the electric vehicle load is pushed into the overnight period (Fig. 3a). Voltages at all houses are kept within required limits (Fig. 3b). Maximum phase unbalance now occurs in the middle of the night, and is slightly reduced with a peak of 3.98% (Fig. 3c).

While the network constraints are not violated, the trade-off of course is that vehicles do not charge as quickly (Fig. 3d):

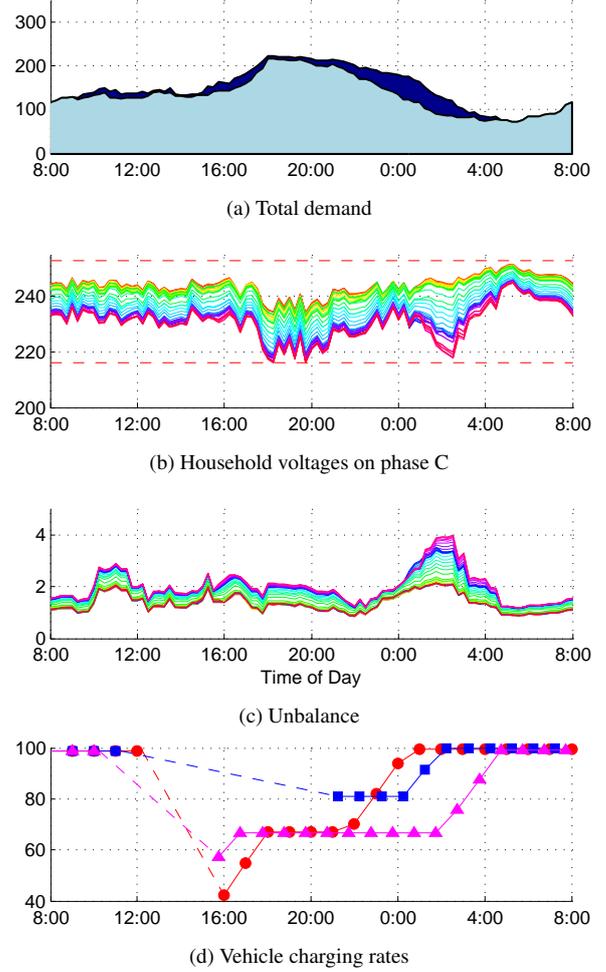


Fig. 3. Optimal charging: vehicle charge rates are chosen using a receding horizon optimal solution. There is almost no increase in peak demand, all houses maintain voltages within required limits, and there is a slight decrease in maximum voltage unbalance. However, vehicle charging is delayed (although vehicles are charged by morning).

- Vehicle 1 arrives home at 16:00 having state of charge 40%. Initially it charges at its maximum rate but as peak demand time begins (18:00) its charging is interrupted. It only starts charging again at 21:00, initially at a reduced rate, and then at its maximum rate, reaching completion at 1:00.
- Vehicle 2 arrives home at 21:00 having a state of charge of 80%. Since there is still much existing household demand, it too must wait until 0:00 to start charging, reaching full state of charge at 2:00.
- Vehicle 3 arrives home at 15:45 having a state of charge of 58%. It charges at its maximum rate initially, but from 16:30 onwards its charging is interrupted. It cannot charge again until 2:00, at which point it charges to completion at 4:45. The reason that Vehicle 3 must wait so long is that it is connected on phase C, the most heavily loaded phase, and it is far from the transformer. Its impact on voltage drop is therefore stronger than most other vehicles, and it must wait longer.

### 4.3 Impact of Constraints

Clearly there are trade-offs involved between the various constraints, and we now explore these in more detail. Unless otherwise specified we are using the objective detailed in Equation (1).

#### Transformer

The real network that our model is based on has a 300kVA transformer, which provides considerable spare capacity. Many networks, however, will have much reduced available capacity. Using our problem formulation, transformer capacity may be shifted up or down and the system responds as required.

Fig. 4 shows the results of modelling the transformer as though it has a more tightly constrained capacity. As can be seen, the choice of transformer capacity can lead to a behaviour similar to water filling: only available capacity up to the threshold is used. The trade-off, of course, is that vehicles may have to wait a long time until they are charged, and even when charging may not get charged as quickly. In the case of a 100kW limit, for example, almost none of the vehicles achieve 100% charge.

However, for a given electric vehicle penetration and known household/vehicle demand, it is possible to determine minimum transformer requirements. In this case, a capacity of 220kW was sufficient to charge all vehicles by 6:00 in the morning while meeting peak household demand.

#### Phase Unbalance

Figure 5 shows unbalance measured in terms of percent deviation of each phase's load from average phase load (left column) and in terms of true voltage unbalance  $|V_-| / |V_+|$  (right column) for varying levels of our unbalance constraint. As we tighten the constraint, the network becomes more and more balanced. In other words, vehicle charge rates are chosen in such a manner that they *rebalance* the network.

The trade-off, once again, is that vehicles (especially those on the more heavily loaded phase) charge less quickly. There are also limits to how much rebalancing is possible; in networks that are more heavily loaded on one phase throughout the day, for example, it is not possible to perfectly rebalance the network as the vehicles on that phase must be charged at some point as well.

#### Voltage Limit

Figure 6 shows the impact of the minimum voltage constraint. Each subfigure shows the voltage at the most sensitive node of the network (the most distant house on the most heavily loaded phase). This network is already quite unbalanced to begin with; as Fig. 6a shows, voltage already approaches the minimum threshold during peak even when there are no vehicles present at all. The impact of this constraint is therefore more immediately apparent in the period from 1:00 to 5:00. With a minimum voltage constraint of 216V (Fig 6b), electric vehicle charging contributes significantly to lowering voltage during this period. As the voltage constraint is increased, first to 222V (Fig. 6c) and then to 226V (Fig. 6d), the vehicle charging rates are reduced to ensure that voltage remains higher. In other words, charging rates are chosen in such a way that electric vehicle charging does not allow voltage at the last house to drop below the specified limit.

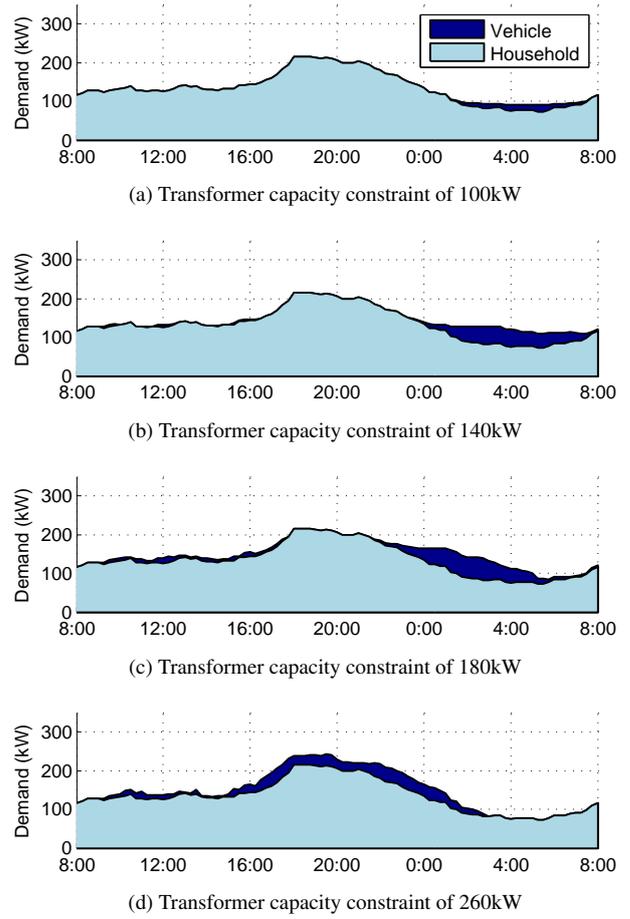


Fig. 4. The effect of the transformer capacity constraint. A tight constraint means that the demand valley is filled, but may not allow all vehicles to charge. A loose constraint leads to an increase in peak demand. The constraint may be chosen so as to make maximum use of available network capacity.

In the examples presented here, the chosen transformer limit (220kW) is the tighter constraint during peak period, so our voltage drop constraint is not as important since charging is limited during peak time already. However, in other networks we have studied, the voltage drop constraint is typically the limiting one.

#### Charging Horizon

Choosing how far to look ahead has a major impact on charging behaviour. A good way to demonstrate this is to examine the price-based objective described in Equation 2. Figure 7 shows the wholesale electricity price for 19/20 June 2012 (our simulated day), as well as the redistribution of electric vehicle load for look-ahead periods of 2, 4, and 6 hours.

A short look-ahead only takes advantage of local minima in the price curve; a longer look-ahead can identify the cheapest time to charge and lead to greater savings. The trade-off once again is that with a longer look-ahead, there is likely to be a greater delay in vehicle charging. It is naturally important to ensure that vehicles are charged at all – the objective in equation 2 does not guarantee this so charge targets are essential to include as an additional constraint. It must also be noted that in this case the solution depends heavily on the accuracy of future price

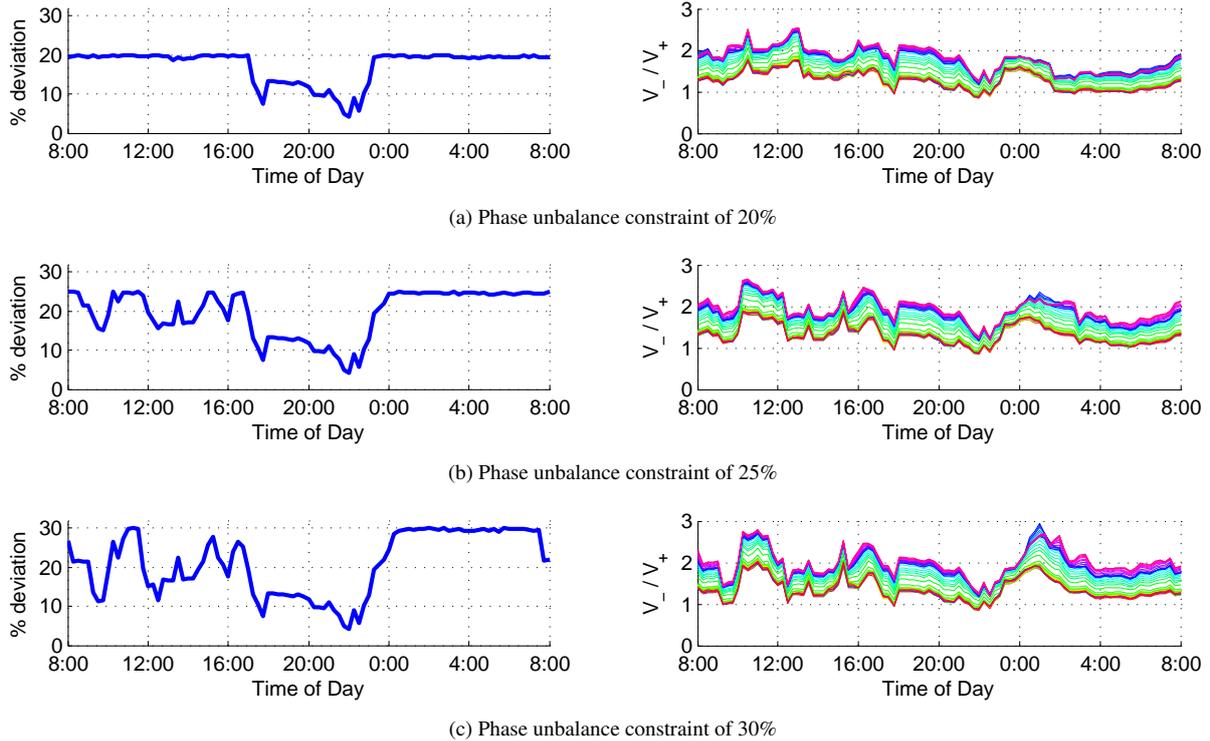


Fig. 5. The effect of the phase unbalance constraint. Phase unbalance is usually measured in terms of  $|V_-| / |V_+|$  (results on right), but our constraint is expressed in terms of maximum percent deviation from average phase load (results on left). With a tight unbalance constraint, true unbalance can be kept low, but vehicles may charge less quickly. With a loose unbalance constraint, vehicle charging increases but so does the network's true unbalance.

prediction. However, for aggregators of vehicle charging that pay a time-varying price, there is a potential for substantial savings using this method.

Note that the system does not perfectly match electric vehicle demand to minimum spot price. This is because there are also other factors at play, such as vehicle charging targets and vehicle arrival / departure, as well as all of the other network constraints involved.

In this case we use historical spot price to demonstrate the method. However, the price-based objective (Equation 2) could be applied in exactly the same way to renewables, such as rooftop PV generation. If a high likelihood of distributed generation can be identified in the future, then vehicle charging can be scheduled to occur during that time using exactly the same method.

## 5. CONCLUSIONS

Electric vehicle charging will have a major impact on distribution networks as EV market share continues to grow. For large scale electric vehicle uptake it will be essential to shift vehicle charging to off-peak periods, in order to avoid massive infrastructure upgrades. In this paper, we have demonstrated how the inherent constraints in the low voltage distribution network may be incorporated into an optimal receding horizon electric vehicle charging solution. In addition, we have shown how these constraints may be used as tools to adjust system performance in any of several desired directions.

A transformer capacity constraint can be used to fill the demand valley and avoid increases in peak demand. A phase unbalance

constraint can be used to enforce system balance, essentially using vehicle charging as a tool to rebalance the system. A voltage limit constraint can ensure that electric vehicles do not contribute to low voltage at any house beyond a chosen limit. Finally, we have demonstrated that the choice of horizon has a significant impact on any objectives that might be time-varying, such as those based on electricity price or availability of distributed generation.

Network control is a very localised problem, with highly variable network configurations and varying standards for safety in practice around the world. We therefore consider the kind of flexibility provided by this tuning of constraints important in the design of solutions that can be taken up on a large scale.

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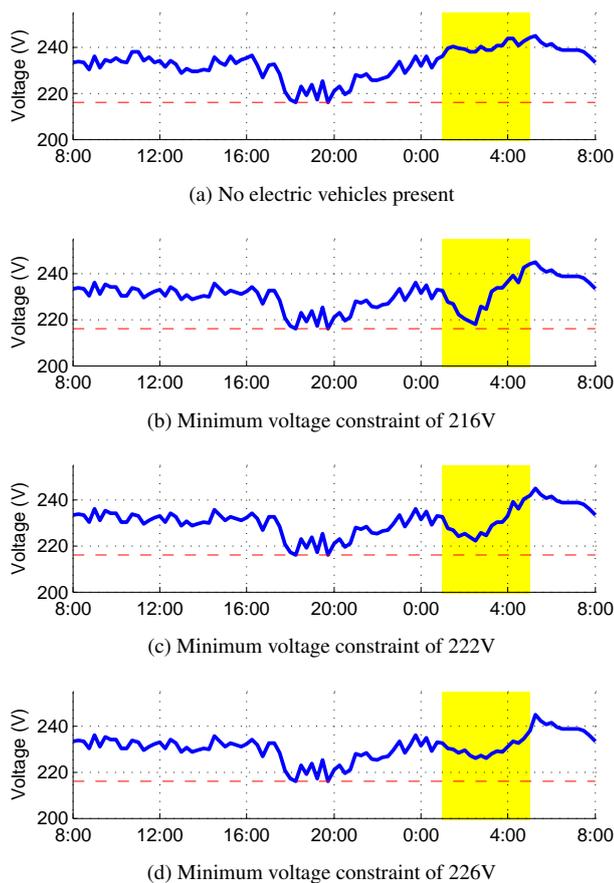


Fig. 6. The effect of the minimum voltage constraint. Each subfigure shows voltage at the most sensitive house in the network. In this case the network already reaches its voltage limitations even when there are no electric vehicles charging, and when there are electric vehicles, the transformer constraint is the limiting factor, not voltage. However, the impact of the voltage constraint is evident during the period 1:00 - 5:00, when vehicle charging rates are chosen in such a way that charging does not contribute to a voltage drop below the constraint limit.

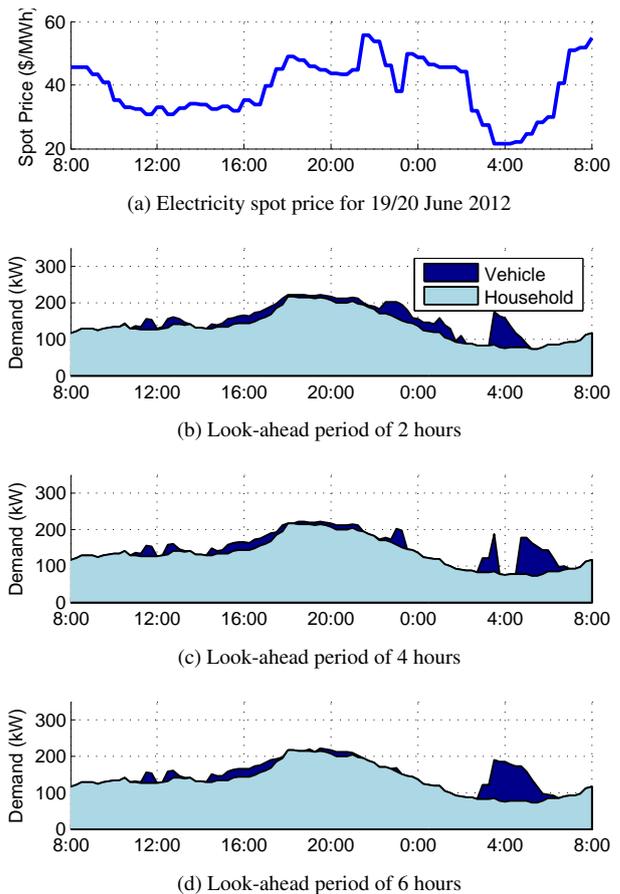


Fig. 7. The effect of the look-ahead constraint for price-based optimisation. With a small look-ahead window, only local minima in the changing price curve are identified. As the look-ahead window is enlarged, the solution examines longer future periods and identifies cheaper times for charging.

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